U N I V E R S I T Y   O F   N I Š

Faculty of Science and Mathematics

XVIII GEOMETRICAL SEMINAR

B O O K   O F   A B S T R A C T S

Vrnjačka Banja, 2014.

CONTENTS

Pablo Alegre, Luis M. Fernández and Alicia Prieto-Martin,
TRANS-S MANIFOLDS **8**

Borysenko Oleksandr Andriyovych,
ISOMETRIC IMMERSION OF KÄHLER SUBMANIFOLDS IN THE FORM OF CONVEX SUBMANIFOLDS IN EUCLIDEAN SPACE **9**

Miroslava Antić and Luc Vrancken,
THREE-DIMENSIONAL MINIMAL CR SUBMANIFOLDS OF THE SPHERE $S^6(1)$ CONTAINED IN A HYPERPLANE **10**

Gülhan AYAR,
A SHUR TYPE THEOREM FOR ALMOST ALPHA-COSYMPLECTIC MANIFOLDS WITH KAEHLERIAN LEAVES **12**

Murat Babaarslan and Yusuf Yayli,
A STUDY ON BERTRAND CURVES AND TIME-LIKE CONSTANT SLOPE SURFACES IN 3-DIMENSIONAL MINKOWSKI SPACE **13**

Vladimir Balan and Jelena Stojanov,
FINSLERIAN-TYPE GAF EXTENSIONS OF THE RIEMANNIAN FRAMEWORK IN DIGITAL IMAGE REGISTRATION **15**

Vitaly Balashchenko,
THE GEOMETRY OF INVARIANT STRUCTURES ON RIEMANNIAN HOMOGENEOUS K-SYMMETRIC SPACES AND NILPOTENT LIE GROUPS ... **16**

Yavuz Balkan,
SOME CURVATURE PROPERTIES OF ALMOST ALPHA-COSYMPLECTIC **F**-MANIFOLDS **18**

Gianluca Bande,
INVARIANT SUBMANIFOLDS OF HERMITIAN BICONTACT STRUCTURES (OR NORMAL METRIC CONTACT PAIRS) **19**

Ergin Bayram and Emin Kasap,
INTRINSIC EQUATIONS FOR A RELAXED ELASTIC LINE OF SECOND KIND IN MINKOWSKI 3-SPACE **20**

Cornelia-Livia Bejan,
ALMOST (PARA)-HERMITIAN MANIFOLDS OF QUASI-CONSTANT (PARA)-HOLOMORPHIC SECTIONAL CURVATURE **21**

BURCU BEKTAŞ, UĞUR DURSUN, ELİF ÖZKARA CANFES
ON TIMELIKE ROTATIONAL SURFACES IN MINKOWSKI SPACE $\mathbf{E}^4_1$ WITH POINTWISE **1**-TYPE GAUSS MAP **22**

Momčilo Bjelica,
COMBINATORIAL BIFURCATIONS OF FLOW ON SIMPLEX EDGES **23**

D. E. Blair,
CONFORMALLY FLAT CONTACT METRIC MANIFOLDS **24**

Beniamino Cappelletti-Montano,
HARD LEFSCHETZ THEOREM FOR SASAKIAN MANIFOLDS **25**

Alfonso Carriazo,
NORMAL APPROXIMATIONS OF REGULAR CURVES AND SURFACES **26**

Uday Chand De,
GENERALIZED QUASI-EINSTEIN MANIFOLDS **27**

Azime Çetinkaya,
GENERALIZED (κ, μ)-CONTACT METRIC MANIFOLDS
WITH RICCI SOLITON **28**

Simona Decu,
ON ISOTROPIC GEOMETRY OF PRODUCTION FUNCTIONS **30**

Ryszard Deszcz,
CURVATURE PROPERTIES OF WARPED PRODUCT MANIFOLDS **31**

Ivko Dimitrić,
CR-SUBMANIFOLDS OF 2-TYPE IN COMPLEX SPACE FORMS **32**

Ivan Dimitrijevic,
COSMOLOGICAL PERTURBATION IN NONLOCAL GRAVITY **33**

Branko Dragović and Nataša Ž. Mišić,
ULTRAMETRICITY, ITS APPLICATIONS AND VISUALIZATION **34**

Irem KUPELİ ERKEN,
PARACONTACT $(\widetilde{\kappa}, \widetilde{\mu})$-MANIFOLDS **35**

Nikolay Yurievich Erokhovets,
G-POLYNOMIAL OF DEFORMATION OF MULTIPLICATION IN A RING **37**

Anatoly Fomenko,
TOPOLOGICAL CLASSIFICATION OF INTEGRABLE SYSTEMS AND BILLIARDS
IN CONFOCAL QUADRICS **38**

Tatiana Fomenko,
ON THE STABILITY PROBLEM OF SEARCH METHODS FOR SINGULARITIES OF
MAPPINGS OF METRIC SPACES **40**

Fabio Gavarini,
AFFINE SUPERGROUPS AND SUPER HARISH-CHANDRA PAIRS **41**

Małgorzata Głogowska,
CURVATURE PROPERTIES OF SOME CLASS OF HYPERSURFACES **42**

Dara Gold,
TOPOLOGICAL CLASSIFICATION OF INTEGRABLE SYSTEMS AND BILLIARDS
IN CONFOCAL QUADRICS **43**

Milica Grbović,
ON GENERALIZED PARTIALLY NULL MANNHEIM CURVES IN MINKOWSKI
SPACE-TIME **44**

Jelena Grujic,
NONSINGULAR BOUNCE COSMOLOGICAL SOLUTIONS IN NONLOCAL GRAVITY **45**

Dmitry V. Gugnin,
ON INTEGRAL COHOMOLOGY RING OF SYMMETRIC PRODUCTS **46**

Sinem Güler and Sezgin Altay Demirbağ,
ON SOME CLASSES OF GENERALIZED QUASI EINSTEIN MANIFOLDS **48**

Graham Hall,
SOME REMARKS ON 4-DIMENSIONAL RICCI-FLAT MANIFOLDS **50**

Irena Hinterleitner and Josef Mikeš,
ON GEODESIC AND HOLOMORPHICALLY PROJECTIVE MAPPINGS **51**

Stefan Ivanov,
THE LICHNEROWICZ-OBATA SPHERE THEOREMS ON A QUATERNIONIC CONTACT MANIFOLD **52**

Włodzimierz Jelonek,
COMPLEX HOLOMORPHIC TOTALLY GEODESIC HOMOTHETIC FOLIATIONS BY CURVES IN KAHLER MANIFOLDS **54**

Nenad Jovanović, Petar Pejić and Sonja Krasić,
THE USE OF VORONOI DIAGRAM IN THE DESIGN OF AN OBJECT CONTAINING A HYPERBOLIC PARABOLOID **55**

Georgios Kaimakamis and Konstantina Panagiotidou,
THE *-RICCI TENSOR OF REAL HYPERSURFACES IN NON-FLAT COMPLEX SPACE FORMS **56**

E.Kantonistova,
TOPOLOGICAL INVARIANTS OF INTEGRABLE HAMILTONIAN SYSTEMS ON THE SURFACES OF REVOLUTION UNDER THE ACTION OF POTENTIAL FIELD **57**

Kamran Khan,
A NOTE ON HEMI-SLANT WARPED PRODUCT SUBMANIFOLDS OF A KAEHLER MANIFOLD **59**

Bahar Kirik and Füsun ÖZEN ZENGİN,
CONFORMAL MAPPINGS OF QUASI-EINSTEIN MANIFOLDS ADMITTING SPECIAL VECTOR FIELDS **60**

Miljan Knežević,
A NOTE ON HARMONIC QUASICONFORMAL MAPPINGS AND SCHWARZ-PICK TYPE INEQUALITIES **61**

Oldrich Kowalski,
DIAGONALIZATION OF THREE-DIMENSIONAL PSEUDO-RIEMANNIAN METRICS **62**

Hristina Krstić, Petar Pejić and Bojana Anđelković,
GEOMETRY OF GOLDEN SECTION AS A BASIS FOR PROTOTYPE OF A HOUSE OF IDEAL PROPORTIONS **63**

Miroslav Kures,
ON THE SEMIHOLONOMIC VELOCITIES AND CONTACT ELEMENTS **64**

Irina Kuzmina,
THE CONFORMAL MODELS OF FIBRATIONS DETERMINED BY THE ALGEBRA OF QUATERNIONS **65**

Tee How Loo,
PSEUDO PARALLEL CR-SUBMANIFOLDS IN COMPLEX SPACE FORMS **66**

Pradip Majhi,
CERTAIN CURVATURE PROPERTIES OF GENERALIZED SASAKIAN SPACE-FORMS **67**

Hristo Manev and Mancho Manev,
THE COMPONENTS OF THE STRUCTURE TENSOR OF ALMOST CONTACT MANIFOLDS WITH B-METRIC IN THE BASIC CLASSES AND THE SMALLEST DIMENSION **68**

Mancho Manev, Stefan Ivanov and Hristo Manev,
SASAKI-LIKE ALMOST CONTACT COMPLEX RIEMANNIAN MANIFOLDS **69**

Veronica Martin-Molina,
ON SOME SPECIAL PARACONTACT METRIC MANIFOLDS **70**

Nikolay Nikolaevich Martynchuk,
SEMI-LOCAL LIOUVILLE EQUIVALENCE OF COMPLEX HAMILTONIAN SYSTEMS DEFINED BY RATIONAL HAMILTONIAN **71**

Koji Matsumoto,
CR-SUBMANIFOLDS WITH THE SYMMETRIC $\nabla\sigma$ IN A LOCALLY CONFORMAL KAEHLER SPACE FORM **72**

Monica Merkle,
ON A VARIATIONAL PROBLEM WITH FREE BOUNDARY ON TWO PARALLEL PLANES **74**

Adela MIHAI, M. Evren AYDIN and Ion MIHAI,
ON SUBMANIFOLDS OF STATISTICAL MANIFOLDS **75**

Ion Mihai,
ON GENERALIZED WINTGEN INEQUALITY **76**

Josef Mikeš and Irena Hinterleitner,
GEODESIC AND HOLOMORPHICALLY PROJECTIVE MAPPINGS **77**

Włodzimierz Mikulski and Jan Kurek,
CALASSICAL LINEAR CONNECTIONS FROM PROJECTABLE ONES ON VERTICAL WEIL BUNDLES **78**

Dmitry Millionshchikov,
COHOMOLOGY OF NILPOTENT LIE ALGEBRAS **79**

Vuk Milošević,
TEMPORARY CHANGES IN GEOMETRY OF MEMBRANE STRUCTURES CAUSED BY LIVE LOADS **80**

Ivan Minchev,
QC HYPERSURFACES IN HYPER-KAEHLER GEOMETRIES **81**

Svetislav Minčić,
ON SOME PROPERTIES OF NON-SYMMETRIC CONNECTIONS **82**

Andrey Mironov,
SEMI-HAMILTONIAN SYSTEM FOR INTEGRABLE GEODESIC OWS ON 2-TORUS **83**

Emil Molnár,
OPTIMAL PACKINGS BY TRANSLATION BALLS IN THE $\widetilde{\mathbf{SL_2R}}$ GEOMETRY **84**

M.N. Mukherjee,
AXIOMATIC CHARACTERIZATION OF QUASI-UNIFORM SPACES AND SOME OF ITS APPLICATIONS **85**

Marian Ioan Munteanu,
ON SOME PERIODIC MAGNETIC CURVES **86**

Pegah MUTLU and Zerrin ŞENTÜRK,
ON LOCALLY CONFORMAL KAEHLER SPACE FORMS **87**

Stanislav Nikolaenko,
ORBITAL EQUIVALENCE OF SOME CLASSICAL INTEGRABLE SYSTEMS **88**

Vladan Nikolić,
POSSIBILITIES OF APPLICATION OF CONES WITH A TORUS KNOT AS DIRECTRIX AND VERTEX ON IT IN THE FORM OF ARCHITECTURAL STRUCTURES **89**

Ana Irina Nistor,
MAGNETIC CURVES IN QUASI-SASAKIAN MANIFOLDS **90**

Zbigniew Olszak,
ON THE CURVATURE OF NULL CURVES IN LORENTZIAN 3-SPACES **91**

Oscar Palmas,
GEOMETRY OF NULL HYPERSURFACES **92**

Anica Pantić,
CURVATURE CONDITIONS ON $\delta(\mathbf{2},\mathbf{2})$ IDEAL SUBMANIFOLDS **93**

Alexander Petkov,
THE SHARP LOWER BOUND OF THE FIRST EIGENVALUE OF THE SUB-LAPLACIAN ON A QUATERNIONIC CONTACT MANIFOLD **94**

Zoran Petrović,
TOPOLOGICAL K-THEORY AND SPACES OF MATRICES **95**

Mileva Prvanović,
BOCHNER-FLAT KÄHLER MANIFOLD AND THE COMPATIBILITY OF RICCI TENSOR **96**

Ljiljana Radović and Slavik Jablan,
ORNAMENTS OF SERBIAN MEDIEVAL FRESCOES **97**

Gabriel Ruiz, Cinthia Barrera and Antonio Di Scala,
CMC CONSTANT ANGLE SURFACES IS SPACE FORMS **98**

Mohammad Shuaib,
SOME WARPED PRODUCT SUBMANIFOLDS OF A KENMOTSU MANIFOLD..**99**

Mića S. Stanković, Milan Lj. Zlatanović, Nenad O. Vesić,
SOME PROPERTIES OF ET-PROJECTIVE TENSORS OBTAINED BY WEYL PROJECTIVE TENSOR..**100**

Sergey Stepanov, Irina Tsyganok and Josef Mikeš,
HODGE-DE RHAM AND TACHIBANA OPERATORS ON COMPACT RIEMANNIAN MANIFOLDS WITH THE BOUNDED POSITIVE AND NEGATIVE CURVATURE OPERATOR OF THE SECOND KIND **101**

E.S. Stepanova and Josef Mikeš,
THE FIELD OF LINEAR ENDOMORPHISMS ATTACHED TO A GEODESIC MAPPING **103**

Milica Stojanović,
SUPERGROUPS OF HYPERBOLIC SPACE GROUPS WITH SIMPLICIAL DOMAINS **104**

Marko Stošić,
COLORED HOMFLY HOMOLOGY OF KNOTS AND LINKS AND GENERALIZED VOLUME CONJECTURE...**105**

T. Šukilović,
GEOMETRY OF 4-DIMENSIONAL NILPOTENT LIE GROUPS WITH NEUTRAL SIGNATURE **106**

Kostadin Trenčevski,
ON THE LINEARLY INDEPENDENT VECTOR FIELDS ON GRASSMANN MANIFOLDS..**107**

Alexey Avgustinovich Tuzhilin,
TITLE MINIMAL SPANNING TREES ON INFINITE SETS **108**

Vladimir Vershinin,
LIE ALGEBRAS OF PURE BRAID GROUPS AND PURE MAPPING CLASS GROUPS...**109**

Andrey Vesnin,
TWO-GENERATED SUBGROUPS OF $\mathbf{PSL(2,C)}$ WHICH ARE EXTREMAL FOR JORGENSEN INEQUALITY AND ITS ANALOGUES.........................**110**

Luc Vrancken, B. Dioos, H. Li and H. Ma,
FLAT ALMOST COMPLEX SURFACES IN THE NEARLY KÄHLER $S^3 \times S^3$ ... **111**

Şerife Nur Yalçın and Ali Çalışkan,
CONNECTION BETWEEN SCREW SYSTEMS AND LINE CONGRUENCE **112**

Handan Yıldırım,
HYPERBOLIC MONGE FORMS IN SENSE OF SLANT GEOMETRY **113**

Milan Lj. Zlatanović and Milica D. Cvetković,
CARTAN'S CONNECTIONS IN A GENERALIZED FINSLER SPACE..........**115**

TRANS-S MANIFOLDS

Pablo Alegre[1], **Luis M. Fernández**[2] **and Alicia Prieto-Martin**[3]
[1]Department of Economics, Cuantitative Methods and Economic History
University Pablo de Olavide, Seville, Spain
[2,3]Department of Geometry and Topology University of Sevilla, Seville, Spain
[1]psalerue@upo.es, [2]lmfer@us.es, [3]aliciaprieto@us.es

Abstract

In contact geometry, J.A. Oubiña introduced the notion of a trans-Sasakian manifold. An almost contact metric manifold $M^{2n+1}(\phi, \xi, \eta)$ is trans- Sasakian if there exist two functions α and β on the manifold such that

$$(\nabla_X \phi)Y = \alpha(g(X,Y)\xi - \eta(Y)X) + \beta(g(\phi X, Y)\xi - \eta(Y)\phi X),$$

for any X, Y on M.
In this communication, we first introduce the notion of a new class of f-metric manifolds: trans-S manifolds; being M^{2n+s} a differentiable manifold with an f structure $(f, \xi_\alpha, \eta_\alpha)$. Hence, we study some of its properties and give some examples.
Finally, we also prove that there are no proper trans-S manifolds in the case of $dim(M) \geq 2n+3$.

AMS Subject Classification: 53C25, 53C40.

ISOMETRIC IMMERSION OF KÄHLER SUBMANIFOLDS IN THE FORM OF CONVEX SUBMANIFOLDS IN EUCLIDEAN SPACE

Borysenko Oleksandr Andriyovych
Professor of Department of Mathematical Analysis and Optimization, Ukraine
aborisenk@gmail.com

Abstract

It has been proved that Kähler convex Submanifolds of low codimension splits as metric product of 2-dimensional convex surfaces in 3-dimensional euclidean Space and euclidean space of small dimension.

THREE-DIMENSIONAL MINIMAL CR SUBMANIFOLDS OF THE SPHERE $S^6(1)$ CONTAINED IN A HYPERPLANE

Miroslava Antić[1] and Luc Vrancken[2]
[1]Faculty of Mathematics, University of Belgrade, Serbia
[2]Université de Valenciennes, France
[1]`mira@math.rs`, [2]`luc.vrancken@univ-valenciennes.fr`

Abstract

It is well known that the sphere $S^6(1)$ admits an almost complex structure J, constructed using the Cayley algebra, which is nearly Kaehler. Let M be a Riemannian submanifold of a manifold $\widetilde{M}$ with an almost complex structure J. It is called a CR submanifold in the sense of Bejancu [2] if there exists a C^∞-differentiable holomorphic distribution $\mathcal{D}_1$ in the tangent bundle such that its orthogonal complement $\mathcal{D}_2$ in the tangent bundle is totally real. If the second fundamental form vanishes on $\mathcal{D}_i$, the submanifold is $\mathcal{D}_i$-geodesic. The first example of a 3-dimensional CR-submanifold was constructed by Sekigawa in [12]. This example was later generalized by Hashimoto and Mashimo in [11]. Both the original example as well as its generalizations are $\mathcal{D}_1$ and $\mathcal{D}_2$-geodesic.

Here, we investigate the class of the three-dimensional minimal CR submanifolds M of the nearly Kaehler 6-sphere $S^6(1)$ which are not linearly full. We show that this class coincides with the class of $\mathcal{D}_1$ and $\mathcal{D}_2$ geodesic CR submanifolds and we obtain a complete classification of such submanifolds. In particular, we show that apart from one special example, the examples of Hashimoto and Mashimo are the only $\mathcal{D}_1$ and $\mathcal{D}_2$-geodesic CR submanifolds.

References

[1] M. Antić, *4-dimensional minimal CR submanifolds of the sphere S^6 contained in a totally geodesic sphere S^5*, J. Geom. Phys, 60 (2010), 96-110.

[2] A. Bejancu, *Geometry of CR-submanifolds,* D. Reidel Publ. Dordrecht, Holland, 1986.

[3] J. Berndt, J. Bolton and L. Woodward, *Almost complex curves and Hopf hypersurfaces in the Nearly Kähler 6-sphere,* Geom. Dedicata, 56 (1995), 237-247.

[4] E. Calabi and H. Gluck, *What are the best almost complex structures on the 6-sphere in Differential Geometry: geometry in mathematical physics and related topics,* Amer. Math. Soc, (1993), 99-106.

[5] B. Y. Chen, *A Riemannian invariant and its applications to submanifold theory,* Results in Math., 27 (1995), 687-696.

[6] B. Y. Chen, *Some pinching and classification theorems for minimal submanifolds,* Archiv. Math. (Basel), 60 (1993), 568-578.

[7] M. Djorić, L. Vrancken, *Three dimensional minimal CR submanifolds in C^6 satisfying Chen's equality,* J. Geom. Phys., 56 (2006), 2279-2288.

[8] J. Erbacher, *Reduction of the codimension of an isometric immersion,* J. Differential Geom., 5 (1971), 333-340.

[9] A. Frölicher, *Zur Differentialgeometrie der komplexen Strukturen,* Math. Ann., 129 (1955), 151-156.

[10] R. Harvey and H. B. Lawson, *Calibrated Geometries,* Acta Math., 148 (1982), 47-157.

[11] H. Hashimoto, K. Mashimo, *On some 3-dimensional CR submanifolds in S^6,* Nagoya Math. J., 156 (1999), 171-185.

[12] K. Sekigawa, *Some CR submanifolds in a 6-dimensional sphere,* Tensor, N. S., 41 (1984), 13-20.

[13] M. Spivak, *A Comprehensive Introduction to Differential Geometry,* Publish or Perish, USA.

[14] R. M. W. Wood, *Framing the exceptional Lie group G_2,* Topology, 15 (1976), 303-320.

A SHUR TYPE THEOREM FOR ALMOST ALPHA-COSYMPLECTIC MANIFOLDS WITH KAEHLERIAN LEAVES

Gülhan AYAR
Yes, Turkey
gulhanayar@gmail.com

Abstract

In this study, we give a shur type therom for almost alpha-cosymplectic manifolds with Kaehlerian leaves. We compute some curvature properties and we obtain phi-holomorphic sectional curvature.

Key words: Cosymplectic manifold, Kaehlerian leaves, Shur theorem.

A STUDY ON BERTRAND CURVES AND TIME-LIKE CONSTANT SLOPE SURFACES IN 3-DIMENSIONAL MINKOWSKI SPACE

Murat Babaarslan[1] **and Yusuf Yayli**[2]

[1]Department of Mathematics, Bozok University, 66100, Yozgat, Turkey

[2]Department of Mathematics, Ankara University, 06100, Ankara, Turkey

[1]murat.babaarslan@bozok.edu.tr, [2]yayli@science.ankara.edu.tr

Abstract

In [2], we defined the notions of Lorentzian Sabban frames and de Sitter evolutes (pseudo-spherical evolutes) of the unit speed time-like curves on de Sitter 2-space $\mathbb{S}_1^2$. We introduced space-like height function of unit speed time-like curves on $\mathbb{S}_1^2$. We studied the invariants and geometric properties of de Sitter evolutes of unit speed time-like curves on $\mathbb{S}_1^2$. Afterwards, we showed that space-like Bertrand curves can be constructed from unit speed time-like curves on $\mathbb{S}_1^2$. We gave the necessary and sufficient conditions for space-like Bertrand curves to be helices. Also we showed that de Sitter Darboux images (pseudo-spherical Darboux images) of space-like Bertrand curves are equal to de Sitter evolutes of unit speed time-like curves on $\mathbb{S}_1^2$. Subsequently we found the relations between space-like Bertrand curves and time-like constant slope surfaces lying in the space-like cone in 3-dimensional Minkowski space $\mathbb{R}_1^3$. In this talk, firstly, we will give similar results for unit speed space-like curves on de Sitter 2-space $\mathbb{S}_1^2$ and hyperbolic space $\mathbb{H}^2$, respectively. Afterwards, we will obtain the relations between Bertrand curves and time-like constant slope surfaces lying in the space-like cone different from above and time-like constant slope surfaces lying in the time-like cone in $\mathbb{R}_1^3$, respectively. To strengthen our main results, we will give some examples and draw the corresponding pictures via *Mathematica* computer program.

AMS Subject Classification: 53B25.

Key words: Lorentzian Sabban frame, pseudo-spherical evolutes, Bertrand curve, pseudo-spherical Darboux images, helix, time-like constant slope surface, de Sitter 2-space, hyperbolic space.

References

[1] Babaarslan, M. and Yayli, Y., *On space-like constant slope surfaces and Bertrand curves in Minkowski 3-space,* Annals of the Alexandru Ioan Cuza University-Mathematics (accepted for publication).

[2] Babaarslan, M. and Yayli, Y., *Time-like constant slope surfaces and space-like Bertrand curves in Minkowski 3-space,* Proceedings of the National Academy of Sciences, India Section A: Physical Sciences (accepted for publication).

[3] Fu, Y. and Yang, D., *On constant slope space-like surfaces in 3-dimensional Minkowski space,* Journal of Mathematical Analysis and Applications, 385, 208-220 (2012).

[4] Fu, Y. and Wang X., *Classification of time-like constant slope surfaces in 3-dimensional Minkowski space,* Results in Mathematics, 63, 1095-1108 (2013).

[5] Izumiya, S. and Takeuchi, N.,*Generic properties of helices and Bertrand curves.* Journal of Geometry 74, 97-109 (2002).

[6] Izumiya, S., Pei, D.H., Sano, T. and Torii, E., *Evolutes of hyperbolic plane curves,* Acta Mathematica Sinica (English Series), 20, 543-550 (2004).

[7] Lopez, R., *Differential geometry of curves and surfaces in Lorentz-Minkowski space,* arXiv:0810.3351v1 [math.DG].

[8] Munteanu, M. I., *From Golden Spirals to Constant Slope Surfaces,* Journal of Mathematical Physics, 51, 1-9, 073507 (2010).

FINSLERIAN-TYPE GAF EXTENSIONS OF THE RIEMANNIAN FRAMEWORK IN DIGITAL IMAGE REGISTRATION

Vladimir Balan[1] **and Jelena Stojanov**[2]
[1]University Politehnica of Bucharest, Romania
[2]University of Novi Sad, Serbia

[1]`vladimir.balan@upb.ro`, [2]`stojanov.jelena@gmail.com`

Abstract

Digital image registration was recently proved to be successfully approached by variational tools which extend the Casseles-Kimmel-Sapiro weighted length problem. Such tools essentilly lead to the so-called Geodesic Active Flow (GAF) process, which relies on the derived mean curvature flow PDE. This prolific process is valuable due to both the provided numeric mathematical insight - which requires specific nontrivial choices for implementing the related algorithms, and the variety of possible underlying specific geometric structures. A natural Finsler extension of Randers type was recently developed by the authors - which emphasizes the anisotropy given by the straightforward gradient, while considering a particular scaling of the Lagrangian. The present work develops to its full extent the GAF process to the Randers Finslerian framework: the evolution equations of the model are determined in detail, Matlab simulations illustrate the obtained theoretic results and conclusive remarks are drawn. Finally, open problems regarding the theoretic model and its applicative efficiency are stated.

THE GEOMETRY OF INVARIANT STRUCTURES ON RIEMANNIAN HOMOGENEOUS K-SYMMETRIC SPACES AND NILPOTENT LIE GROUPS

Vitaly Balashchenko
Belarusian State University, Belarus
balashchenko@bsu.by

Abstract

It is known that distributions generated by almost product structures are applicable, in particular, to some problems in the theory of Monge-Ampère equations [1]. We characterize canonical distributions defined by canonical almost product structures on Riemannian homogeneous k-symmetric spaces in the sense of types $\boldsymbol{AF}$ (anti-foliation), $\boldsymbol{F}$ (foliation), $\boldsymbol{TGF}$ (totally geodesic foliation). Algebraic criteria for all these types on k-symmetric spaces of orders $k = 4, 5, 6$ were obtained. Note that canonical distributions on homogeneous k-symmetric spaces are closely related to special canonical almost complex structures and f-structures, which were recently applied by I. Khemar [2] to studying elliptic integrable systems.

Further, we construct and study (jointly P.Dubovik) metric left-invariant f-structures on solvable and nilpotent Lie groups relating to the generalized Hermitian geometry (e.g. nearly Kähler and Hermitian f-structures). In this sense some filiform Lie groups are of special interest. We also study left-invariant f-structures on 2-step and some other nilpotent Lie groups represented as Riemannian homogeneous k-symmetric spaces. Some general and particular examples including Heisenberg groups and their generalizations are considered.

Finally, we show that so-called "metallic" structures (golden, silver and others) recently introduced in [3] can be effectively realized using the theory of canonical structures on homogeneous k-symmetric spaces [4].

References

[1] A. Kushner, *Almost product structures and Monge-Ampèr equations.* Lobachevskii J. Math. 2006. V. 23. P. 151–181.

[2] I. Khemar, *Elliptic integrable systems: a comprehensive geometric interpretation.* Memoirs of the AMS. 2012. V. 219, no. 1031. x+217 pp.

[3] C.-E. Hretcanu, M. Crasmareanu, *Metallic structures on Riemannian manifolds.* Revista de la Union Matematica Argentina. 2013. V. 54, no. 2. P. 15–27.

[4] V.V. Balashchenko, Yu.G. Nikonorov, E.D. Rodionov, V.V. Slavsky, *Homogeneous spaces: theory and applications: monograph,* Polygrafist, Hanty-Mansijsk, 2008 (in Russian). - http://elib.bsu.by/handle/123456789/9818

SOME CURVATURE PROPERTIES OF ALMOST ALPHA-COSYMPLECTIC F-MANIFOLDS

Yavuz Balkan
Düzce University, Faculty of Arts and Sciences, Department of Mathematics, Düzce, Turkey
y.selimbalkan@gmail.com

Abstract

In this study, we consider almost alpha-cosymplectic f-manifolds and we get Riemannian curvatures properties. We compute some sectional curvatures and scalar curvature. Finally, we give two examples to clarify some our results.

Key words: cosymplectic manifolds, almost alpha-cosymplectic manifolds, f-manifolds.

INVARIANT SUBMANIFOLDS OF HERMITIAN BICONTACT STRUCTURES (OR NORMAL METRIC CONTACT PAIRS)

Gianluca Bande
Università di Cagliari, Italy
gbande@unica.it

Abstract

We discuss some recent results concerning the invariant submanifolds of a normal metric contact pair (MCP) M with decomposable structure tensor ϕ (or Hermitian bicontact structure). By invariance of the submanifolds we mean invariance either by ϕ or by the two natural complex structures associated to M. In particular we prove that their minimality is related to the two Reeb vector fields associated to the MCP.

Key words: Contact Pair, Vaisman manifolds, invariant submanifold, minimal submanifold.

INTRINSIC EQUATIONS FOR A RELAXED ELASTIC LINE OF SECOND KIND IN MINKOWSKI 3-SPACE

Ergin Bayram[1] **and Emin Kasap**[2]
[1,2]Ondokuz Mayis University, Faculty of Arts and Sciences,
Mathematics Deparment, Samsun, Turkey
[1]erginbayram@yahoo.com, [2]kasape@omu.edu.tr

Abstract

Let $\alpha(s)$ be an arc on a connected oriented surface S in Minkowski 3-space, parametrized by arc length s, with torsion τ and length l. The total square torsion T of $\alpha(s)$ is defined by $T = \int_0^l \tau^2 ds$. The arc $\alpha(s)$ is called a relaxed elastic line of second kind if it is an extremal for the variational problem of minimizing the value of T within the family of all arcs of length l on S having the same initial point and initial direction as $\alpha(s)$. In this study, we obtain differential equation and boundary conditions for a relaxed elastic line of second kind on an oriented surface in Minkowski 3-space.

Key words: Calculus of variations, Minkowski 3-space, Elastic line.

ALMOST (PARA)-HERMITIAN MANIFOLDS OF QUASI-CONSTANT (PARA)-HOLOMORPHIC SECTIONAL CURVATURE

Cornelia-Livia Bejan
Department of Mathematics, Technical University "Gh. Asachi", Iasi, Romania
bejanliv@yahoo.com

Abstract

Some results obtained by Mileva Prvanovic on almost Hermitian manifolds of constant sectional curvature are extended here to the quasi-constant sectional curvature on one side and to the almost para-Hermitian manifolds on the other side. The basic tool used here is the holomorphic curvature tensor field. Some examples are provided.

ON TIMELIKE ROTATIONAL SURFACES IN MINKOWSKI SPACE $\mathbb{E}_1^4$ WITH POINTWISE 1-TYPE GAUSS MAP

[1]BURCU BEKTAŞ, [2]UĞUR DURSUN,
[3]ELİF ÖZKARA CANFES
[1,2,3] Istanbul Technical University, Turkey
[1]bektasbu@itu.edu.tr, [2]udursun@itu.edu.tr, [3]canfes@itu.edu.tr

Abstract

In this work, we study timelike rotational surfaces in the Minkowski space $\mathbb{E}_1^4$ with pointwise 1-type Gauss map. First we determine all timelike rotational surfaces in $\mathbb{E}_1^4$ with meridian curves lying in 2-dimensional timelike plane and having pointwise 1-type Gauss map of the first kind. Then we obtain all flat timelike rotational surfaces of elliptic, hyperbolic or parabolic type in $\mathbb{E}_1^4$ with 2-dimensional axis and having pointwise 1-type Gauss map.

AMS Subject Classification: 53B25, 53C50.

Key words: Pointwise 1-type Gauss map, rotational surfaces, parallel mean curvature vector, normal bundle.

COMBINATORIAL BIFURCATIONS OF FLOW ON SIMPLEX EDGES

Momčilo Bjelica
Technical faculty “Mihajlo Pupin”, University of Novi Sad, Zrenjanin, Serbia
bjelica@tfzr.uns.ac.rs

Abstract

In this paper is given a proof of Fibonacci type recurrence

$$S_0 = \left(\frac{1}{d-1}, 0, \ldots, 0\right), \quad S_1 = (0, 1, 0, \ldots, 0), \quad S_2 = (0, 0, 1, \ldots, 1),$$

$$S_{n+1} = (d-1)^2 S_{n-2} + (d-2) S_n, \qquad n \geq 3.$$

Initial flow on one edge of d-dimensional simplex has $(d-1)$-furcation at the incoming vertex. Propagation of the flow continues under the same condiitons. S_n is the number of scintillations of simplex vertices in n-th iteration.

AMS Subject Classification: 05C05, 52A20, 92D20.

Key words: flow, tree on comlete graph, Fibonacci, gene codes.

References

[1] Bjelica, M., *Matrix representation of tetrahedral flows,* Proceedings of Mathematical and Informatical Technologies MIT 2011, August 27-31. Vrnjačka Banja 2011., Serbia, pp. 40-43.

CONFORMALLY FLAT CONTACT METRIC MANIFOLDS

D. E. Blair
Department of Mathematics, Michigan State University,
East Lansing, MI 48824-1027, USA

blair@math.msu.edu

Abstract

We will begin with a review of the basic ideas of contact manifolds and associated metrics. Our first topic will then be to discuss the question of constant curvature for contact metric manifolds, a question which has a very short answer. The main topic of the lecture is whether or not there exist conformally flat, contact metric manifolds which are not of constant curvature. In dimensions ≥ 5 this is open, but in dimension 3, these exist. We will also relate the 3-dimensional examples constructed to a problem in astrophysics.

HARD LEFSCHETZ THEOREM FOR SASAKIAN MANIFOLDS

Beniamino Cappelletti-Montano
University of Cagliari, Italy

b.cappellettimontano@unica.it

Abstract

The celebrated Hard Lefschetz Theorem ([2], [4]) states that in any compact Kähler manifold the exterior multiplication bysuitable powers of the symplectic form induces isomorphisms between the de Rham cohomology spaces of complementary degrees. This gives a strong topologicalobstruction for a compact symplectic manifold to carry a Kähler structure. In this talk I shall present the recent joint work [1] with A. De Nicola and I. Yudin (University of Coimbra, Portugal) showing that similar isomorphisms do existalso for compact Sasakian manifolds, though, differently from Kähler geometry, the exterior multiplications by $d\eta$ or by $\eta \wedge d\eta$ do notmap harmonic forms into harmonic forms, η being the contact form. Such isomorphisms are proven to be independent of the choice of any compatibleSasakian metric on a given contact manifold. As a consequence, an obstruction for a contact manifold to admit a compatible Sasakian metric is found. In the final part of the talk I shall mention the very recent paper [3] of Li, who applying the above Hard Lefschetz Theorem found examples of simplyconnected K-contact manifolds without any Sasakian structures in any dimension ≥ 9.

Key words: contact manifold, K-contact, Sasakian, Hard Lefschetz Theorem.

References

[1] B. Cappelletti-Montano, A. De Nicola, I. Yudin, *Hard Lefschetz Theorem for Sasakian manifolds*, arXiv:1306.2896, 2013.

[2] S. Lefschetz, *L'analysis situs et la géométrie algébrique*, Gauthiers-Villars, Paris, 1924.

[3] Yi Lin, *Lefschetz contact manifolds and odd dimensional symplectic geometry*, arXiv:1311.1431, 2013.

[4] W. V. D. Hodge, *The theory and applications of harmonic integrals*, 2nd ed., Cambridge University Press, London, 1952.

NORMAL APPROXIMATIONS OF REGULAR CURVES AND SURFACES

Alfonso Carriazo
University of Seville, Spain
carriazo@us.es

Abstract

Bezier curves and surfaces are a very useful tool in Geometric Modeling, with many applications. In this talk, we will offer a new method to provide approximations of regular curves and surfaces by Bezier ones, with the corresponding examples.

GENERALIZED QUASI-EINSTEIN MANIFOLDS

Uday Chand De
Department of Pure Mathematics University of Calcutta 35
Ballygaunge Circular Road Kolkata 700019 West Bengal, India

uc_de@yahoo.com

Abstract

Qusi-Einstein manifold is a natural generalization of Einstein manifold. Quasi Einstein manifolds arose during the study of exact solutions of the Einstein eld equations as well as during considerations of quasi-umbilical hypersurfaces of Euclidean spaces. For instance, the Robertson-Walker spacetime are quasi Einstein manifolds. Also, quasi Einstein manifold can be taken as a model of the perfect fluid spacetime in general relativity. So quasi-Einstein manifolds have some importance in the general theory of relativity. Quasi-Einstein manifold is denoted by $(QE)_n$. On the other hand a 2-quasi umbilical hypersurface of a Euclidean space or a space of constant curvature gives rise to a certain generalization of qusi-Einstein manifold called generalized quasi-Einstein manifold $G(QE)_n$. We obtain some geometric properties of $G(QE)_n$. Several examples of $G(QE)_n$ have been constructed to prove the existence of such a manifold.

Key words: Quasi-Einstein manifold, Generalized Quasi-Einstein manifold, Space of Constant curvature, 2-Quasi umbilical hyperfurface.

GENERALIZED (κ,μ)-CONTACT METRIC MANIFOLDS WITH RICCI SOLITON

Azime Çetinkaya
Piri Reis University, Turkey
azzimece@hotmail.com

Abstract

The object of the present paper is to the study generalized (κ,μ)-contact metric manifolds admitting Ricci soliton. First I investigate the conditions $R(X,\xi).P = 0$, $R(X,\xi).\widetilde{C} = 0$, $R(X,\xi).S = 0$ and $P(X,\xi).S = 0$ on generalized (κ,μ)-contact metric manifolds given with Ricci soliton where R is Riemannian curvature tensor, P is projective curvature tensor, $\widetilde{C}$ is quasiconformal curvature tensor and S is Ricci tensor. Then I give an example for 3-dimensional generalized (κ,μ)-contact metric manifolds and an example for 3-dimensional generalized (κ,μ)-contact metric manifolds with Ricci soliton.

AMS Subject Classification: 53A30, 53C15, 53D10.

Key words: generalized (κ,μ)-contact metric manifold, Ricci soliton.

References

[1] Arnold V. I.,*Contact geometry: the geometrical method of Gibb's thermodynamics,* Proceed-ings of the Gibb's Symp., Yale University (May 15-17, 1989), American Math. Soc., (1990), 163-179.

[2] Bagawadi C.S. and Ingalahalli G., *Certain Results on Ricci Solitons in Trans-Sasakian Manifolds,* Hindawi Publishing Corporation Journal of Mathematics, 2013.

[3] Blair D. E ., *Contact manifolds in Riemannian geometry,* Lecture Notes in Mathematics Vol. 509,, Springer-Verlag, Berlin-New York, (1976).

[4] Blair D. E. and Oubina J. A., *Conformal and related changes of metric on the product of two almost contact metric manifolds,* Publications Matematiques, 34(1990), 199-207.

[5] Calin C., Crasmareanu M., *From the Einshart problem to Ricci solitons in f-Kenmotsu manifolds,* Bull. Malays. Math. Soc. 33(2010), 361-368.

[6] Chave T., Valent G., *Quasi-Einstein metrics and their renoirmalizability properties,* Helv. Phys. Acta. 69(1996), 344-347.

[7] Chave T., Valent G., *On a class of compact and non-compact quasi-Einstein metrics and their renoirmalizability properties,* Nuclear Phys. B, 478(1996), 758-778.

[8] Cho J.T., Kimura M., *Ricci solitons and Real Hypersurfaces in a complex space form,* Tohoku Math. J., 61(2009), 205-212.

[9] Chow B., Kno D., *The Ricci flow: An introduction,* Mathematical Surveys and Monographs 110, American Math. Soc., (2004).

[10] De U. C. and Mondal A. K., *On 3-dimensional almost contact metric manifolds satisfying certain curvature conditions,* Com. Kor. Math. Soc., 24(2009) 265-275.

[11] Geiges H., *A brief history of contact geometry and topology, Expo. Math., 19(2001), 25-53.*

[12] Hamilton R. S., *The Ricci flow on surfaces,* Mathematics and general relativity (Santa Cruz,CA, 1986), 237-262, Contemp. Math. 71, American Math. Soc.,(1988).

[13] Ivey T., *Ricci solitons on compact 3-manifolds,* Differential Geo. Appl. 3(1993), 301-307.

[14] Janssens D., Vanhecke L., *Almost contact structures and curvature tensors,* Kodai Math. J.,4(1)(1981), 1-27.

[15] Koufogiorgos T. and Tsichlias C., *Generalized* (κ,μ)*-contact metric manifolds with* $\kappa grad\kappa = constant$*,* J. Geom. 78 (2003), 83-91.

[16] Koufogiorgos T. and Tsichlias C., *On the existence of a new class of contact metric manifolds,* Canad. Math. Bull. XX(Y) (2000), 1-8.

[17] Maclane S., *Geometrical mechanics II,* Lecture notes, University of Chicago, (1968).

ON ISOTROPIC GEOMETRY OF PRODUCTION FUNCTIONS

Simona Decu
SIAT, Bucharest, Romania
simona.decu@gmail.com

Abstract

In economics, the production function is one of the key notions of neoclassical theories. Identification and analysis of production functions are fundamental economic approaches. Basic studies are focused on the Cobb-Douglas and CES production functions identified as production hypersurfaces in Euclidean spaces. More other general researches are conducted afterwards. Our objective is to study the production functions via isotropic geometry.

CURVATURE PROPERTIES OF WARPED PRODUCT MANIFOLDS

Ryszard Deszcz
Wrocław Univeristy of Environmental and Life Sciences, Wrocław, Poland
Ryszard.Deszcz@up.wroc.pl

Abstract

We present results on warped product manifolds of dimension $\geqslant 4$ satisfying some pseudosymmetry type curvature conditions. Our talk bases on [1]-[5].

Key words: warped product manifolds, pseudosymmetry type curvature conditions.

References

[1] J. Chojnacka-Dulas, R. Deszcz, M. Głogowska and M. Prvanović, *On warped products manifolds satisfying some curvature conditions,* J. Geom. Phys. **74** (2013), 328–341.

[2] R. Deszcz, M. Hotloś, J. Jełowicki, H. Kundu, and A.A. Shaikh, *Curvature properties of Gödel metric,* Int. J. Geom. Meth. Modern Phys. **11** (2014), 1450025 (20 pages).

[3] R. Deszcz and D. Kowalczyk, *On some class of pseudosymmetric warped products,* Colloq. Math. **97** (2003), 7–22.

[4] R. Deszcz, M. Plaue and M. Scherfner, *On Roter type warped products with 1-dimensional fibres,* J. Geom. Phys. **69** (2013), 1–11.

[5] R. Deszcz and M. Scherfner, *On a particular class of warped products with fibres locally isometric to generalized Cartan hypersurfaces,* Colloq. Math. **109** (2007), 13–29.

CR-SUBMANIFOLDS OF 2-TYPE IN COMPLEX SPACE FORMS

Ivko Dimitrić
Pennsylvania State University Fayette,
The Eberly Campus, 1 University Drive, Uniontown, PA 15401, USA
ivko@psu.edu

Abstract

We characterize certain classes of CR-submanifolds of complex projective and complex hyperbolic spaces that are of low Chen type (primarily 2-type) in a suitable pseudo-Euclidean space of complex matrices, embedded by projection operators. These characterizations use conditions involving well known intrinsic and extrinsic objects such as the shape operators, Ricci tensor, etc. We also prove some nonexistence results for some families of 2-type CR-submanifolds of complex space forms. For example, there exist no 2-type holomorphic submanifolds of a complex hyperbolic space via the embedding by projectors, which contrasts with the situation in a complex projective space, where there are such examples. Further it is shown that there are no ruled submanifolds of 2-type in a non-flat complex space form. Some of these results come from a joint work with M. Djorić.

COSMOLOGICAL PERTURBATION
IN NONLOCAL GRAVITY

(joint work with Branko Dragovich, Jelena Grujic and Zoran Rakic)

Ivan Dimitrijevic
Faculty of Mathematics, University of Belgrade, Studentski trg 16, Belgrade, Serbia
ivand@matf.bg.ac.rs

Abstract

After discovery of accelerating expansion of the Universe, there has been a renewed interest in gravity modification. One of promising approaches is nonlocal modification with the Ricci curvature R in the action replaced by a suitable function $F(R,\Box)$, where $\Box = \frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu$ is the Laplace-Beltrami operator. In particular we analyze the modification in the form

$$S = \int \sqrt{-g}\Big(\frac{R}{16\pi G} + R^{-1}\mathcal{F}(\Box)R\Big)d^4x,$$

where $\mathcal{F}(\Box)$ is an analytic function. Also we will discuss perturbations of this model with de Sitter background.

ULTRAMETRICITY, ITS APPLICATIONS AND VISUALIZATION

Branko Dragovich[1] **and Nataša Ž. Mišić**
[1]Institute of Physics, University of Belgrade, Belgrade, Serbia
[1]dragovich@ipb.ac.rs

Abstract

Ultrametricity is a mathematical concept related to ultrametric (non-Archimedean) spaces, where distances satisfy strong triangle inequality. Ultrametric spaces have some very unusual properties from the point of view of our standard experience. Nevertheless, there are many examples of ultrametric spaces in mathematics, physics, biology, linguistics,... In fact, all systems with hierarchical structure have some utrametric properties. The most advanced examples of ultrametricity are based on p-adic numbers. There are more and more applications of ultrametricity, which include p-adic analysis in mathematics, spin glasses in physics, string theory and cosmology, p-adic mathematical physics, adelic models, taxonomy, phylogenetics, protein dynamics, the genetic code in biology,... Visualization of ultrametricity is very important for its understanding and it is usually done by various trees, fractals and the Cantor set. In this contribution, we mainly use p-adic ultrametricity in modeling the genetic code.

PARACONTACT $(\widetilde{\kappa},\widetilde{\mu})$-MANIFOLDS

Irem KUPELİ ERKEN
Art and Science Faculty, Department of Mathematics,
Uludag University, 16059 Bursa, Turkey

iremkupeli@uludag.edu.tr

Abstract

We study paracontact metric manifolds for which the Reeb vector field of the underlying contact structure satisfies a nullity condition ($\widetilde{R}(X,Y)\xi = \widetilde{\kappa}(\eta(Y)X - \eta(X)Y) + \widetilde{\mu}(\eta(Y)\widetilde{h}X - \eta(X)\widetilde{h}Y)$ for some real numbers $\widetilde{\kappa}$ and $\widetilde{\mu}$). This class of pseudo-Riemannian manifolds, which includes para-Sasakian manifolds, was recently defined in [8]. In this paper we show in fact that there is a kind of duality between those manifolds and contact metric (κ,μ)-spaces. In particular, we prove that, under some natural assumption, any such paracontact metric manifold admits a compatible contact metric (κ,μ)-structure (eventually Sasakian).

Moreover, we prove that the nullity condition is invariant under D-homothetic deformations and determines the whole curvature tensor field completely. Finally non-trivial examples in any dimension are presented and the many differences with the contact metric case, due to the non-positive definiteness of the metric, are discussed.

References

[1] Blair, D. E., *Riemannian geometry of contact and symplectic manifolds, Second edition,* Progress in Mathematics, 203, Birkhauser, Boston, 2010.

[2] Blair, D. E., Koufogiorgos, T., Papantoniou, B. J., *Contact metric manifolds satisfying a nullity condition,* Israel J. Math. 91 (1995), 189-214.

[3] Boeckx, E., *A full classification of contact metric (κ,μ)-spaces,* Illinois J. Math. 44 (2000), 212-219.

[4] Cappelletti Montano, B., *Bi-Legendrian connections,* Ann. Polon. Math. 86 (2005), 79-95.

[5] Cappelletti Montano, B., *The foliated structure of contact metric (κ,μ)-spaces,* Illinois J. Math. 53 (2009), 1157-1172.

[6]] Cappelletti Montano, B., *Bi-paracontact structures and Legendre foliations,* Math. J. 33 (2010), 473-512.

[7] Cappelletti Montano, B., Di Terlizzi, L., *Contact metric (κ,μ)-spaces as bi-Legendrian manifolds,* Bull. Aust. Math. Soc. 77 (2008), 373-386.

[8] Cappelletti Montano, B., Di Terlizzi, L., *Geometric structures associated to a contact metric (κ, μ)-space,* Pacific J. Math. 246 no. 2 (2010), 257-292.

[9] Pang, M. Y. *The structure of Legendre foliations,* Trans. Amer. Math. Soc. 320 n.2 (1990), 417-453.

[10] Zamkovoy, S. *Canonical connections on paracontact manifolds,* Ann. Glob. Anal. Geom. 36 (2009), 37-60.

G-POLYNOMIAL OF DEFORMATION OF MULTIPLICATION IN A RING

Nikolay Yurievich Erokhovets
Lomonosov Moscow State University, Russian Federation
erochovetsn@hotmail.com

Abstract

We will give the construction of a homomorphism that arises from a deformation of multiplication in a graded ring that in a special case of ring of polytopes gives the toric g-polynomial of Stanley and in the case of quasi-symmetric functions gives the g-polynomial of Billera, Hsiao ans van Willigenburg. These objects are defined inductively and are not so easy to handle with. Our result gives purely algebraic construction and in future may lead to generalized g- or h-polynomial of a convex polytope and g-polynomial of a stably complex manifold.

Let $\mathbb{A}$ be a commutative associative ring with a unit and $A = \sum A^{2n}$, $n \geqslant 0$, be a connected graded associative $\mathbb{A}$-algebra. Consider a graded commutative multiplicative semigroup S with a unit such that $\deg s = 2k > 0$ for any $s \neq 1$. Let $S = S_1 \sqcup S_2$, $S_1 \cap S_2 = \{1\}$ be its decomposition into two subsemigroups. Denote by $A[S]$ the graded ring which is a free module over A with basis S, that is $A[S]$ consists of all finite linear combinations $\sum a_s s$. Degree of a monomial $a_s s$ is given by $\deg a_s + \deg s$, and the multiplication is defined by the rule $(a_s s)(a_t t) = (a_s a_t)(st)$ and linearity. For example, $S = \{\alpha^k t^l\}$, and $S_1 = \{\alpha_k t^l | k > l\}$, $S_2 = \{\alpha^k t^l | k \geqslant l\}$.

Deformation of multiplication in A is a ring homomorphism $\Psi \colon A \to A[S]$ such that $\Psi(a) = a \cdot 1 + \sum_{s \neq 1} a_s s$. We will use the fact that any ring homomorphism $A \to R$ can be uniquely extended to the ring homomorphism $A[S] \to R[S]$.

Theorem. *Let $Psi \colon A \to A[S]$ be a deformation of the multiplication in the $\mathbb{A}$-algebra A. Then*

1) There exists a unique pair of $\mathbb{A}$-linear maps $\widetilde{G} \colon A \to \mathbb{A}[S_1]$ and $G \colon A \to \mathbb{A}[S_2]$, such that $\widetilde{G}(1) = G(1) = 1$ and $\widetilde{G} = G\Psi$.

2) Both $\widetilde{G}$ and G are ring homomorphisms.

The proof genelaizes ideas by Billera, Hsiao and van Willigengurg, and by Buchstaber and the author.

Key words: deformation of multiplication, toric g-polynomial, polytopes, quasi-symmetric functions.

TOPOLOGICAL CLASSIFICATION OF INTEGRABLE SYSTEMS AND BILLIARDS IN CONFOCAL QUADRICS

Anatoly Fomenko
Faculty of Mathematics and Mechanics, Moscow State University, Moscow, Russia
atfomenko@mail.ru

Abstract

The theory of invariants based on the topological approach was suggested by A. T. Fomenko and developed by A.T.Fomenko, H.Zieschang, A. V. Bolsinov, and others for the study of integrable Hamiltonian systems with two degrees of freedom. This theory allows to investigate different qualitative properties of such systems and to conclude whether two systems are equivalent (in some sense) or not. Primarily, we mean the following three types of equivalence: Liouville equivalence, topological and smooth orbital equivalence. For each type of equivalence, a non-degenerate integrable system restricted to a 3-dimensional isoenergetic surface is assigned with a discrete invariant (molecule) which is a graph with some numerical marks. The main result of the theory can now be formulated in the following way: two integrable Hamiltonian systems considered on non-degenerate isoenergy 3-surfaces are equivalent (in one of the senses mentioned) if and only if the corresponding molecules are the same. In particular, two integrable non-degenerate systems are Liouville equivalent on 3-dimensional isoenergy sufaces if and only of their Fomenko-Zieschang invariants (graphs with numerical marks r,n,e) are the same. V.Dragovich and M.Radnovich calculated these marks for some billiard systems in the 2-dimensional domain bounded by confocal quadrics. The work was continued by V.Fokicheva, who did the calculations of Fomenko-Zieschag invariants for so called "covering integrable billiards". The latter notion was introduced by A.Oshemkov and E.Kudryavtseva. Let us say, that the flat 2-dimensional domain Ω bounded by quadrics from the continuous family of quadrics (with parameter Λ) is called equivalent to the domain Ω' bounded by quadrics from the same family, iff Ω' is obtained from Ω by symmetries via axes and/or continuous change of parameter Λ with the only condition: Λ does not coincide with b. Really, the flat billiard systems admit the following generalization. For example, consider k copies of the domain bounded by two confocal ellipses, and make a cut along the lower segment of the coordinate line O_y. Then glue cuts by the following rule: the left edge of the cut on the i-th copy is glued to the right edge of the cut on the $i+1$-th copy. This domain is called Δk. If we glue the rest of the edges of the cut together we

get the new domain. If we consider generalization of this construction for another types on flat confocal billiards (in domains with angles), then we obtain a new large class of billards in “covering domains with angles”. V.Fokicheva has obtained the topological classification of such domains and also the Liouville classification of corresponding Hamiltonian integrable billiard systems.

References

[1] A. T. Fomenko, H. Zieschang, *A topological invariant and a criterion for the equivalence of integrable Hamiltonian systems with two degrees of freedom,* Math. USSR-Izv., 36:3 (1991), 567-596.

[2] V. Dragovic, M. Radnovic, *Bifurcations of Liouville tori in elliptical billiards,* Regul.Chaotic Dyn. 2009. 14, No. 4-5. 479-494.

[3] V.V.Fokicheva, *Description the topology of the Hamiltonian integrable system "billiard within an ellipse,* Vestn. Moscow. University.Math. Mech. 2012, v. 5,p. 31-35.

[4] V.V.Fokicheva, *Description the topology of the Hamiltonian integrable system "billiard in an domain bounded by the segments of the confocal quadrics,* Vestn. Moscow. University.Math.Mech. (In print).

ON THE STABILITY PROBLEM OF SEARCH METHODS FOR SINGULARITIES OF MAPPINGS OF METRIC SPACES

Tatiana Fomenko
Lomonosov Moscow State University, Russia
tn-fomenko@yandex.ru

Abstract

Given a finite collection of (one-valued or multivalued) mappings between metric spaces, the issues of stability of the searching and approximation for some their singularities are considered. Here we mean by singularities some special subsets such as common fixed point set, coincidence set, the set of common preimages of a given closed subspace, set of common roots, concerning a given finite collection of mappings. Some new results will be presented concerning different statements of the stability problem. The presented results are based of the use of so called (α, β)-search functionals $(0 < \alpha < \beta)$ and functionals strictly subjected to convergent series, which were recently introduced by the author.

References

[1] Fomenko T.N., *Stability of Cascade Search,* Izvestiya: Mathematics, 74:5, 2010. Pp. 1051-1068.

[2] Fomenko T.N., *Cascade search: Stability of reachable limit points,* Moscow University Mathematics Bulletin, vol.65, Number 5/October, 2010. Pp.179-185.

[3] Fomenko T.N., *Functionals strictly subordinate to Series and Search for Solutions of Equations,* Doklady Mathematics, 2013, vol.88, Number 3, pp.748-750.

AFFINE SUPERGROUPS AND SUPER HARISH-CHANDRA PAIRS

Fabio Gavarini
Università di Roma "Tor Vergata" - dipartimento di Matematica, Italy
gavarini@axp.mat.uniroma2.it

Abstract

I present a new method to study affine supergroups (in the algebraic super-geometry setting) by means of super Harish-Chandra pairs. Namely, there exists a well known, natural functor from affine supergroups to super Harish-Chandra pairs. Conversely, I provide an explicit, functorial construction which, with each super Harish-Chandra pair, associates an affine supergroup that is "globally split". In fact, if we restrict to globally split affine sueprgroups then the two functors are quasi-inverse to each other. so they provide equivalences between the category of globally split affine supergroups and that of super Harish-Chandra pairs.

Such a result was known in other contexts (the smoth differential and the complex analytic one), or in some special cases. I extend it to the algebro-geometric framework, with a totally different, more geometrical method - by bare hands, somehow.

Key words: Algebraic Supergroups, Lie superalgebras, super Harish-Chandra pairs.

CURVATURE PROPERTIES OF SOME CLASS OF HYPERSURFACES

Małgorzata Głogowska
Department of Mathematics
Wrocław Univeristy of Environmental and Life Sciences, Wrocław, Poland
Magorzata.Glogowska@up.wroc.pl

Abstract

We present results on quasi-Einstein Cartan type hypersurfaces isometrically immersed in spaces of constant curvature. Our talk bases on [1]-[3].

Key words: hypersurfaces, quasi-Einstein manifolds, pseudosymmetry type curvature conditions.

References

[1] R. Deszcz, M. Głogowska, H. Hashiguchi, M. Hotloś and M. Yawata, *On semi-Riemannian manifolds satisfying some conformally invariant curvature condition*, Colloq. Math. **131** (2013), 149–170.

[2] R. Deszcz, M. Hotloś and Z. Şentürk, *On curvature properties of certain quasi-Einstein hypersurfaces*, Int. J. Math. **23** (2012), 1250073 (17 pages).

[3] M. Głogowska, *On quasi-Einstein Cartan type hypersurfaces*, J. Geom. Phys. **58** (2008), 599–614.

TOPOLOGICAL CLASSIFICATION OF INTEGRABLE SYSTEMS AND BILLIARDS IN CONFOCAL QUADRICS

Dara Gold
Boston University, USA
daracaseygold@gmail.com

Abstract

When fitting a finite set of points in R^N with a k-dimensional manifold we want to both minimize the manifold's distance from the set of points and its curvature. This requires a penalty function $C(\phi)$ from the space of all embeddings (ϕ) (of a fixed k-dimensional manifold into R^N) into the real numbers which incorporates both a distance and curvature term. The goal is to find a minimum of this function, giving an ideal manifold embedding. This amounts to performing gradient flow of the penalty function C in the space of embeddings to flow to minimal ϕ values. Problems that arise include the fact that the space of embeddings is open in the space of all maps, allowing the flow to leave the space of embeddings. Another open question is how long the flow must occur to get within epsilon distance of minimal values.

ON GENERALIZED PARTIALLY NULL MANNHEIM CURVES IN MINKOWSKI SPACE-TIME

Milica Grbović
Faculty of Sience, University of Kragujevac, Serbia
milica_grbovic@yahoo.com

Abstract

We define generalized partially null Mannheim curve α in Minkowski space-time. We consider the cases when the generalized Mannheim mate curves of α are the spacelike, the timelike, null, pseudo null and partially null curves. We prove that there are no generalized partially null Mannheim curves whose generalized Mannheim mate curves are the null and the pseudo null curves. In all other cases, we determine generalized partially null Mannheim curve α and its mate curve.

NONSINGULAR BOUNCE COSMOLOGICAL SOLUTIONS IN NONLOCAL GRAVITY

(joint work with I. Dimitrijevic, B. Dragovich and Z. Rakic)

Jelena Grujic
Teachers Training Faculty, University of Belgrade,
Kraljice Natalije 43, Belgrade, Serbia

jelenagg@gmail.com

Abstract

In this contribution we consider nonlocal gravity action without matter in the form

$$S = \int d^4x\sqrt{-g}\Big(\frac{R-2\Lambda}{16\pi G} + R^p\mathcal{F}(\Box)R\Big),$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of the d'Alembertian $\Box$ and $p = +1, -1$. We present a few $a(t)$ nonsingular bounce cosmological solutions for the above two actions, see references [1-5].

References

[1] T. Biswas, T. Koivisto, A. Mazumdar, *Towards a resolution of the cosmological singularity in non-local higher derivative theories of gravity,* JCAP **1011** (2010) 008 [arXiv:1005.0590v2 [hep-th]].

[2] A. S. Koshelev, S. Yu. Vernov, *On bouncing solutions in non-local gravity,* Phys. Part. Nuclei **43**, 666–668 (2012)[arXiv:1202.1289v1 [hep-th]].

[3] T. Biswas, A. S. Koshelev, A. Mazumdar, S. Yu. Vernov, *Stable bounce and inflation in non-local higher derivative cosmology,* JCAP **08** (2012) 024, [arXiv:1206.6374v2 [astro-ph.CO]].

[4] I. Dimitrijevic, B. Dragovich, J. Grujic, Z. Rakic, *New cosmological solutions in non-local modified gravity,* Rom. Journ. Phys. **58** (5-6), 550–559 (2013) [arXiv:1302.2794 [gr-qc]].

[5] I. Dimitrijevic, B. Dragovich, J. Grujic, Z. Rakic, *A new model of nonlocal modified gravity,* Publications de l'Institut Mathematique **94** (108), 187–196 (2013).

ON INTEGRAL COHOMOLOGY RING OF SYMMETRIC PRODUCTS

Dmitry V. Gugnin
Moscow State University, Assistant Prof., Ph.D., Moscow, Russia
dmitry-gugnin@yandex.ru

Abstract

Let us denote by $\mathrm{Sym}^n X$ the n-th symmetric product X^n/S_n of a topological space X. The famous result of A.Dold'58 [1] states that if two connected CW complexes X and Y has equal integral homology $H_i(X;\mathbb{Z}) = H_i(Y;\mathbb{Z}), 1 \le i \le q$, then $H_i(\mathrm{Sym}^n X;\mathbb{Z}) = H_i(\mathrm{Sym}^n Y;\mathbb{Z})$, $1 \le i \le q$, for all $n > 1$.

From this moment suppose that all spaces X has finitely generated integral homology in each dimension. Then from A.Dold's result it is easy to check that integral cohomology $H^*(\mathrm{Sym}^n X;\mathbb{Z})$ is also finitely generated in each dimension. Then one has ring isomorphisms

$$H^*(\mathrm{Sym}^n X;\mathbb{Z}) \otimes \mathbb{Q} \cong (H^*(\mathrm{Sym}^n X;\mathbb{Z})/\mathrm{Tor}) \otimes \mathbb{Q} \cong H^*(\mathrm{Sym}^n X;\mathbb{Q}).$$

If for two given CW complexes X and Y their rational cohomology rings are equal, $H^*(X;\mathbb{Q}) = H^*(Y;\mathbb{Q}) = A^*$, then by simple classical Transfer Theorem one has equality $H^*(\mathrm{Sym}^n X;\mathbb{Q}) = H^*(\mathrm{Sym}^n Y;\mathbb{Q}) = S^n A^*$, where $S^n A^* := (A^{\otimes n})^{S_n}$.

The first main result of my talk is the following theorem.

Theorem 1(G.,2014). *Let X and Y are connected CW complexes such that $H^*(X;\mathbb{Z})/\mathrm{Tor} \cong H^*(Y;\mathbb{Z})/\mathrm{Tor}$. Then there exists an isomorphism of rings $H^*(\mathrm{Sym}^n X;\mathbb{Z})/\mathrm{Tor} \cong H^*(\mathrm{Sym}^n Y;\mathbb{Z})/\mathrm{Tor}$ for all $n > 1$.*
Moreover, for any additive basis

$$a_{i,j} \in H^i(X;\mathbb{Z})/\mathrm{Tor}, i > 0, 1 \le j \le \mathrm{rank}(H^i(X;\mathbb{Z})/\mathrm{Tor}),$$

and integral multiplication table $a_{i,j}a_{k,l} = c^{s,t}_{i,j;k,l} a_{s,t}$, there is an explicit algorithm for constructing some additive basis of $H^*(\mathrm{Sym}^n X;\mathbb{Z})/\mathrm{Tor}$, $n > 1$, and computing the multiplication table for this basis.

It is not hard to prove that for any k-dimensional topological manifold M^k and any $n > 1$ the space $\mathrm{Sym}^n M^k$ is a manifold without boundary **iff** $k = 2$. For $k = 1$ the space $\mathrm{Sym}^n M^1$ is an n-dimensional manifold with boundary, and for $k > 2$ the space $\mathrm{Sym}^n M^k$ is not even a homology manifold (in some points it has local homology distinct from the local homology of $\mathbb{R}^{nk}$).

So, the most interesting case is $\mathrm{Sym}^n M^2$ for 2-dimensional manifolds M^2. It is easy to check that $\mathrm{Sym}^n M^2$ is orientable **iff** M^2 is orientable. The classical fact from algebraic geometry states that $\mathrm{Sym}^n M_g^2$, $n > 1$, has a canonical structure of smooth projective variety for any compact Riemann surface M_g^2 of arbitrary genus $g \geq 0$. It is easy to prove that $\mathrm{Sym}^n(\mathbb{C}P^1) = \mathbb{C}P^n$. So, the case $g = 0$ is trivial.

Suppose $g > 0$. The famous result of I.G.Macdonald'62 [2] states the following.

Theorem (I.G.Macdonald,1962). *Suppose M_g^2 is an arbitrary compact Riemann surface of genus $g > 0$. Then the ring $H^*(\mathrm{Sym}^n M_g^2; \mathbb{Z})$ has no torsion and is isomorphic to free graded commutative algebra over $\mathbb{Z}$ on $2g$ generators of degree 1 and one generator of degree 2 factorized by some concrete integral relations (I.G.Macdonald's ideal).*

But, as was noticed by the author in 2012, the proof of I.G.Macdonald's theorem contains three serious gaps. The elimination of these gaps is the second main result to be presented on my talk. The first and the second gaps are topological in nature, and they are eliminated by theorem 1 above. The third gap is eliminated by purely algebraic reasons.

The author is grateful to the Corresponding Member of the Russian Academy of Sciences Professor V. M. Buchstaber, to Professor A. A. Gaifullin and Professor T. E. Panov for valuable discussions.

AMS Subject Classification: 55S15, 57N65.

Key words: integral cohomology ring, symmetric products, Riemann surface.

References

[1] Dold, A., *Homology of symmetric products and other functors of complexes,* Ann. of Math. 1958. Vol. 68. P. 54-80.

[2] Macdonald, I. G., *Symmetric products of an algebraic curve,* Topology. 1962. Vol. 1. P. 319-343.

ON SOME CLASSES OF GENERALIZED QUASI EINSTEIN MANIFOLDS

Sinem Güler[1] **and Sezgin Altay Demirbağ**[2]
[1] Istanbul Technical University, Turkey
[2] Istanbul Technical University, Turkey

[1]singuler@itu.edu.tr, [2]saltay@itu.edu.tr

Abstract

The object of the present paper is to study the generalized quasi Einstein manifolds introduced by M.C. Chaki in 2001 [1] satisfying some conditions. The notion of quasi Einstein manifold has been first introduced by M.C. Chaki and R.K. Maity [2]. For a long time, quasi Einstein manifolds have been attracting the attention of researches. First of all, due to its application in mechanics, in particular, in fluid mechanics, and also as generalization of Einstein spaces, i.e., spaces where the Einstein tensor is zero [3-5]. We meet quasi Einstein manifolds during the study of exact solutions of Einstein's field equation. Thus the study of generalized quasi Einstein manifolds becomes meaningful due to its applications in the general theory of relativity and cosmology. 4-dimensional quasi Einstein spacetime represents a perfect fluid spacetime in cosmology [5]. The importance of a generalized quasi Einstein manifold lies in the fact that such 4-dimensional semi-Riemannian mani! fold is relavant to the study of general relativistic fluid spacetime admitting heat flux [6]. The global properties of a such spacetime is under investigation.

Firstly, we consider the condition $R.S = 0$ on a generalized quasi Einstein manifold, when R and S denote the curvature tensor and the Ricci tensor of the manifold, respectively and we obtain a sufficient condition for a generalized quasi Einstein manifold to be a quasi Einstein manifold. Then we find every Ricci semi symmetric generalized quasi Einstein manifold is nearly quasi Einstein manifold [7]. Later, we prove that a conformally flat Ricci semi symmetric generalized quasi Einstein manifold has a proper concircular vector field and such a manifold is a subprojective manifold in the sense of Kagan and also is a warped product $I \times_{e^q} M^*$, where (M^*, g^*) is an $(n-1)$-dimensional Riemannian manifold.

AMS Subject Classification: 53C15, 53C25, 53B15, 53B20.

Key words: Generalized quasi Einstein manifold, Ricci semi symmetric manifold, concircular vector field, subprojective space, warped product space.

References

[1] Chaki M.C., *On Generalized quasi-Einstein manifolds*, Publ. Math. Debrecen, 58, 638-691, 2001.

[2] Chaki M.C., and Maity R.K., *On-quasi Einstein Manifolds*, Publ. Math. Debrecen, 57, 297-306, 2000.

[3] Petrov A.Z., *New Methods in the General Theory of Relativity*, , in Russian Nauka, Moscow, 1966.

[4] Deszcz R, Dillen F., Verstraelen, L. and Vrancken L., *Quasi-Einstein totally real submanifolds of the Kahler 6-Sphere*, Tohoku. Math. J., 51, 461-478, 1999.

[5] O'Neill B., *Semi-Riemannian Geometry* , Academic Press, 1983.

[6] Ray D., *Gödel-like cosmological solutions*, J. Math. Phys., 21 2797, 1980.

[7] Gazi, A.K., and De, U.C., *On the Existence of Nearly Quasi-Einstein Manifolds*, Navi Sad J. Math. 39 (2), 111-117, 2009.

SOME REMARKS ON 4-DIMENSIONAL RICCI-FLAT MANIFOLDS

Graham Hall
Institute of Mathematics, University of Aberdeen,
Aberdeen AB24 3UE, Scotland, United Kingdom

g.hall@abdn.ac.uk

Abstract

This talk will deal with 4-dimensional Ricci-flat manifolds admitting a metric of arbitrary signature. The main idea is to consider the metric, its associated Levi-Civita connection, its Riemann curvature tensor and its sectional curvature function and to show that, with a few, rather special exceptions in some cases, they are equivalent, that is, each of these geometrical objects essentially determines the others. Further comments will be made in this direction regarding the Weyl conformal and projective tensors. The techniques used will involve some holonomy theory and a consideration of the subalgebras of the orthogonal groups $o(4)$, $o(1,3)$ and $o(2,2)$. Some remarks will be made on the case when the metric has Lorentz signature and on its consequences for Einstein's general theory of relativity.

ON GEODESIC AND HOLOMORPHICALLY PROJECTIVE MAPPINGS

Irena Hinterleitner[1] **and Josef Mikeš**[2]
[1]Brno University of Technology, Czech Republic
[2]Palacky University Olomouc, Czech Republic
[1]`hinterleitner.irena@seznam.cz`, [2]`josef.mikes@upol.cz`

Abstract

We was proved that geodesic and holomorphically projective mappings of (pseudo-) Riemannian manifolds preserve the class of differentiability. Also, if the Einstein space admits a nontrivial geodesic mapping onto a (pseudo-) Riemannian manifold V, then V is an Einstein space. If a four-dimensional Einstein space with non-constant curvature globally admits a geodesic mapping f onto a (pseudo-) Riemannian manifold V with differentiability metric, then the mapping f is affine and, moreover, if the scalar curvature is non-vanishing, then the mapping is homothetic.

THE LICHNEROWICZ-OBATA SPHERE THEOREMS ON A QUATERNIONIC CONTACT MANIFOLD

Stefan Ivanov
University of Sofia "St. Kliment Ohridski", Faculty of Mathematics and Informatics,
Blvd. James Baucher 5, 1164 Sofia, Bulgaria
ivanovsp@fmi.uni-sofia.bg

Abstract

Lichnerowcz [5] showed that on a compact Riemannian manifold (M, h) of dimension n for which the Ricci curvature is greater than or equal to that of the round unit n-dimensional sphere $S^n(1)$, the first positive eigenvalue λ_1 of the Laplace operator is greater than or equal to the first eigenvalue of the sphere, $\lambda_1 \geq n$. Subsequently Obata [6] proved that equality is achieved if and only if the Riemannian manifold is isometric to $S^n(1)$ by noting that the trace-free part of the Riemannian Hessian of an eigenfunction f with eigenvalue $\lambda = n$ vanishes.

We prove quaternionic contact versions of the Lichnerowicz's and Obata's sphere theorems.

Theorem 0.1 ([2]) *Let $(M, g, \mathbf{Q})$ be a compact quaternionic contact (qc) manifold of dimension $4n+3 > 7$. If for some positive constant k_0 the Ricci tensor and the torsion of the Biquard connection ∇ satisfy*

$$Ric(X,X)+\frac{2(4n+5)}{2n+1}T^0(X,X)+\frac{6(2n^2+5n-1)}{(n-1)(2n+1)}U(X,X) \geq k_0 g(X,X), \quad X \in H, \tag{1}$$

then the first nonzero eigenvalue λ_1 of the sub-laplacian $\triangle f$ obeys the inequality $\lambda_1 \geq \frac{n}{n+2}k_0$.

Theorem 0.2 ([3]) *Let $(M, \eta, g, \mathbb{Q})$ be a compact qc manifold of dimension bigger than seven whose Ricci tensor satisfies* (1)*. Then the first positive eigenvalue of the sub-Laplacian takes the smallest possible value, $\lambda_1 = \frac{n}{n+2}k_0$, if and only if the qc manifold is qc equivalent to the standard 3-Sasakian sphere.*

Theorem 0.2 follows from the following

Theorem 0.3 ([3]) *Let $(M, \eta, g, \mathbb{Q})$ be a qc manifold of dimension $4n+3 > 7$ which is complete with respect to the associated Riemannian metric $h = g + (\eta_1)^2 + (\eta_2)^2 + (\eta_3)^2$ and ∇ be the Biquard connection.*

Suppose there exists a non-constant smooth function f whose horizontal Hessian satisfies

$$\nabla df(X,Y) = -fg(X,Y) - \sum_{s=1}^{3} df(\xi_s)\omega_s(X,Y). \tag{2}$$

Then the qc manifold $(M, \eta, g, \mathbb{Q})$ is qc homothetic to the unit (4n+3)-dimensional 3-Sasakian sphere.

The work relies on considerations in [1]. The proof of Theorem 0.1 follows from a qc version of the Bochner formula. We achieve the proof of Theorem 0.3 by showing first that M is isometric to the unit sphere S^{4n+3} and then that M is qc-equivalent to the standard 3-Sasakian structure on S^{4n+3}. To this effect we show that (2) forces the torsion of the Biquard connection vanishes leading the Riemannian Hessian to be trace-free and the classical Obata theorem implies that M is isometric to the unit sphere. In order to prove the qc-equivalence part we show that the qc-conformal curvature vanishes, which gives the local qc conformal equivalence with the 3-Sasakian sphere due to [4, Theorem 1.3], and then prove a Liouville-type result leading the existence of a global qc-conformal map between M and the 3-Sasakian sphere.

References

[1] Ivanov, S., Minchev, I., & Vassilev, D., *Quaternionic contact Einstein structures and the quaternionic contact Yamabe problem*, Memoirs Am. Math. Soc., vol. 231, n. 1086.

[2] Ivanov, S., Petkov, A., & Vassilev, D., *The sharp lower bound of the first eigenvalue of the sub-Laplacian on a quaternionic contact manifold*, J. Geom. Anal., DOI 10.1007/s12220-012-9354-9;

[3] ———*The Obata sphere theorems on a quaternionic contact manifold of dimension bigger than seven*, arXiv:1303.0409, submitted.

[4] Ivanov, S., & Vassilev, D., *Conformal quaternionic contact curvature and the local sphere theorem*, J. Math. Pures Appl. **93** (2010), 277–307.

[5] Lichnerowicz, A., *Géométrie des groupes de transformations.* Travaux et Recherches Mathematiques, III. Dunod, Paris 1958.

[6] Obata, M., *Certain conditions for a Riemannian manifold to be iosometric with a sphere.* J. Math. Soc. Japan **14**, no.3, (1962), 333–340.

COMPLEX HOLOMORPHIC TOTALLY GEODESIC HOMOTHETIC FOLIATIONS BY CURVES IN KAHLER MANIFOLDS

Włodzimierz Jelonek
Cracow University of Technology, Poland
wjelon@pk.edu.pl

Abstract

We give a complete classification of complex holomorphic totally geodesic homothetic foliations by curves in compact Kahler manifolds. We prove that the leaves of such foliation are fibers of holomorphic CP^1-bundle over Hodge manifold. We present examples of such foliations in various geometric problems.

THE USE OF VORONOI DIAGRAM IN THE DESIGN OF AN OBJECT CONTAINING A HYPERBOLIC PARABOLOID

Nenad Jovanović[1], Petar Pejić[2] and Sonja Krasić[3]
[1,2,3]Faculty of Civil Engineering and Architecture, University of Niš, Serbia
[1]nelejovanovic@yahoo.com, [2]petar.pejic@gaf.ni.ac.rs,
[3]sonja.krasic@gaf.ni.ac.rs

Abstract

In this paper, we will discuss the Voronoi diagram and Voronoi cells that have the potential broad application in architecture. The aim of this paper is to show the possibilities of Voronoi diagrams and grid-shell structures in the design of doubly ruled surface and comparison with the same surface executed in reinforced concrete shell system. As an example we used the famous building designed by architect Felix Candela, "Los Manantiales", and compared it to a model derived in the software package "Rhinoceros" and add-on "Voronax". Result of the paper is to present structural and economic effectiveness of constructing buildings in the grid-shell system using the Voronoi diagram.

Key words: Voronoi diagram, Hyperbolic paraboloid, doubly ruled surface, "Rhinoceros", "Voronax".

THE *-RICCI TENSOR OF REAL HYPERSURFACES IN NON-FLAT COMPLEX SPACE FORMS

Georgios Kaimakamis[1] and Konstantina Panagiotidou[2]
[1]Hellenic Military Academy, Attiki, Greece
[2]Aristotle University of Thessaloniki, Greece
[1]gmiamis@gmail.com, [2]kapanagi@gen.auth.gr

Abstract

The complex projective space $\mathbb{C}P^n$ and the complex hyperbolic space $\mathbb{C}H^n$ are called non-flat complex space forms, when it is not necessary to be distinguished. Let M be a real hypersurface in a non-flat complex space form. Many geometers, such as Berndt, Kim, Ortega, Pérez, Santos, Suh, Takagi etc have contributed in the study of real hypersurfaces in non-flat complex space forms in terms of their operators (shape operator, structure Jacobi operator) and their tensor field (Ricci tensor).

In this talk, results concerning real hypersurfaces in non-flat complex space forms in terms of their *-**Ricci tensor**, S^*, which was first studied by Hamada ([1]), will be presented. More precisely, the non-existence of real hypersurfaces in non-flat complex space forms with parallel *-Ricci tensor, i.e. $\nabla_X S^* = 0$ for any $X \in TM$, will be discussed ([2]). Furthermore, other types of parallelism of *-Ricci tensor such as semi-parallelism, i.e. $R \cdot S^* = 0$, pseudo-parallelism, i.e. $R(X,Y) \cdot S^* = L\{(X \wedge Y) \cdot S^*\}$ with $L \neq 0$ and conclusions for real hypersurfaces satisfying the previous conditions will be provided ([3]). Finally, a new type of Ricci soliton the *-**Ricci soliton** and results concerning real hypersurfaces admitting this new type will be developed ([4]).

Key words: Real hypersurface, Parallel, *-Ricci tensor, Complex projective plane, Complex hyperbolic plane.

References

[1] T. Hamada, “Real hypersurfaces of complex space forms in terms of Ricci *-tensor”, *Tokyo J. Math.* **25** (2002), 473-483.

[2] G. Kaimakamis and K. Panagiotidou, “Parallel *-Ricci tensor of real hypersurfaces in $\mathbb{C}P^2$ and $\mathbb{C}H^2$”, arxiv:1401.6794v1.

[3] G. Kaimakamis and K. Panagiotidou, “Conditions of parallelism of *-Ricci tensor of real hypersurfaces in $\mathbb{C}P^2$ and $\mathbb{C}H^2$”, preprint.

[4] G. Kaimakamis and K. Panagiotidou, “*-Ricci solitons of real hypersurfaces in non-flat complex space forms”, preprint.

TOPOLOGICAL INVARIANTS OF INTEGRABLE HAMILTONIAN SYSTEMS ON THE SURFACES OF REVOLUTION UNDER THE ACTION OF POTENTIAL FIELD

E.Kantonistova
Moscow State Lomonosov University, Russia

kysin@rambler.ru

Abstract

Consider a manifold S, which is diffeomorphic to $(a,b) \times S$ (a and b are finite numbers), with metrics $ds^2 = dr^2 + f^2(r)d\varphi^2$ in polar coordinates $(r, \varphi(mod 2\pi))$. Functions $V(r)$, $f(r)$ are smooth functions on (a,b), and $f(r) > 0$ on (a,b). We call the function $V(r)$ a *potential function*, and $f(r)$ — a *function of revolution*. A system defined by pair of functions $(f(r), V(r))$ is an integrable Hamiltonian system. Let us call it a *system on the surface of revolution*.

Its phase space has a dimension of 4 (it has a coordinates $(r, \varphi, p_r, p_\varphi)$).

The system has two integrals of motion: H — the energy, and p_φ — the projection of momentum on axis of revolution. The Hamilton function of this system has a form

$$H = \frac{p_r^2}{2} + \frac{p_\varphi^2}{2f^2(r)} + V(r).$$

Definition. We call the map $\Phi: S \to \mathbb{R}^2$: $(r, \varphi, p_r, p_\varphi) \mapsto (H(r, \varphi, p_r, p_\varphi), p_\varphi(r, \varphi, p_r, p_\varphi))$ the *momentum map*.

Definition. If $rkd\Phi(x) < 2$, then x is called a *singular point*, and $\Phi(x)$ is called a *singular value*. The set Σ of singular points is called a *bifurcation diagram*.

Assume that we have constructed a bifurcation diagram for some system, i.e. we have a set of curves on the plane with coordinates (H, p_φ). Let us fix an arbitrary value of $H = H_0$. Then a level submanifold of the phase space, which corresponds to $H = H_0$, is a manifold of dimension 3, and it is called an *isoenergetic manifold* Q_H^3. Moreover, to any point (H_0, p_φ), which does not belong to the curves of bifurcation diagram, correspond one or more Liouville tori lying in Q_H^3, and to any point (H_0, p_φ), which belongs to the bifurcation curve, correspond some bifurcations of tori. The bifurcation of a certain type is called the *atom*.

For each value of H we can construct a graph with vertices corresponding to the bifurcations of tori and the ribs corresponding to the

regular values of momentum map. This graph is called a *Fomenko–Ziechang invariant* (or simply a *molecule*).

Theorem 1. *If $f(r)$ and $V(r)$ are smooth functions on (a, b), then the system $(f(r), V(r))$ has only atoms of type A and B.*

Theorem 2. *All the marks of type "r" on the ribs of molecule have a value 0 or ∞ for investigated systems.*

Moreover, all Fomenko–Ziechang invariants were calculated (I will speak about it more detailly during my talk).

There exist another invariant — the invariant of trajectory equivalence. It is called a *function (and vector) of rotation* (it has not the same meaning that a function $f(r)$ has). You can read the theory about it in [1].

The functions of rotation for the investigated systems were calculated. And particularly was proved Fomenko hypothesis for the gravitational potential $V(r) = r$:

Fomenko hypothesis. *For two systems on the surface of revolution, given by the pair $(f(r), V(r))$, there exist a value $\widetilde{H}$, such that Fomenko–Ziechang invariants and vectors of rotation on its ribs coincide for any $H_1, H_2 : \ H_i > \widetilde{H}, \ i = 1, 2$.*

References

[1] A.V.Bolsinov, A.T.Fomenko, *Integrable Hamiltonian systems*, Izhevsk: Izd.dom "Udmurtskij universitet", 1999.

A NOTE ON HEMI-SLANT WARPED PRODUCT SUBMANIFOLDS OF A KAEHLER MANIFOLD

Kamran Khan
Department of Mathematics, Aligarh Muslim University, Aligarh-202002 India
kamran35d@gmail.com

Abstract

Many differential geometric properties of submanifolds of a Kaehler manifold are conceived via canonical structure tensors P and F on the submanifold. For instance, a CR-submanifold of a Kaehler manifold is a CR-product (i.e. locally a Riemannian product of a holomorphic and a totally real submanifold) if and only if P is parallel on the submanifold. Since, warped products are generalized version of Riemannian product of manifolds, we consider the covariant derivatives of the structure tensors on a hemi-slant submanifold of a Kaehler manifold. Our investigations have led us to establish characterizations of hemi-slant submanifolds. Under the conditions obtained in Theorems 4.5, 4.7 and Corolloary 4.6, the submanifold reduces to a warped product submanifold.

Key words: Warped product, hemi-slant submanifolds, Kaehler manifold

CONFORMAL MAPPINGS OF QUASI-EINSTEIN MANIFOLDS ADMITTING SPECIAL VECTOR FIELDS

Bahar Kirik[1] **and Füsun ÖZEN ZENGİN**[2]
[1,2]Istanbul Technical University, Faculty of Science and Letters, Department of Mathematics, Istanbul, Turkey
[1]bkirik@itu.edu.tr,[2]fozen@itu.edu.tr

Abstract

As it is known, Einstein manifolds play an important role in geometry as well as in general relativity. Einstein manifolds form a natural subclass of the class of quasi-Einstein manifold. In this work, we investigate conformal mappings of quasi-Einstein manifolds. Considering this mapping, we examine some properties of these manifolds. After that, we also study some special vector fields under this mapping of these manifolds and some theorems about them are proved.

A NOTE ON HARMONIC QUASICONFORMAL MAPPINGS AND SCHWARZ-PICK TYPE INEQUALITIES

Miljan Knežević
Faculty of Mathematics, University of Belgrade, Serbia
kmiljan@matf.bg.ac.rs

Abstract

We will give a short review of harmonic quasiconformal mappings theory and its relation with hyperbolic geometry. Also, we will consider which properties of HQC mappings and of hyperbolic metric are essential for validity of some Schwarz-Pick type inequalities.

DIAGONALIZATION OF THREE-DIMENSIONAL PSEUDO-RIEMANNIAN METRICS

(joint work with M. Sekizawa)

Oldrich Kowalski
Matematický ústav UK, Praha, Czech Republic
kowalski@karlin.mff.cuni.cz

Abstract

We prove that every analytic Riemannian or Lorentzian metric in R^3 can be locally expressed in the diagonal form.

AMS Subject Classification: 53B30, 53C21, 53C50.

Key words: pseudo-Riemannian metric, PDE methods, Cauchy-Kovalevski Theorem.

GEOMETRY OF GOLDEN SECTION AS A BASIS FOR PROTOTYPE OF A HOUSE OF IDEAL PROPORTIONS

Hristina Krstić[1], Petar Pejić[2] and Bojana Anđelković[3]
[1,2,3]Faculty of Civil Engineering and Architecture, University of Niš, Serbia
[1]hristinaa@hotmail.com, [2]petar.pejic@gaf.ni.ac.rs

Abstract

Architecture is considered as a mixture of two totally opposite fields -mathematics and art, and as a link between them, there is geometry. The architecture has always searched for the perfect dimensions and ideal proportions, and golden section is one of the mentioned options. Is the ratio of the edges in golden rectangle really ideal as it is considered throughout history, is the question which will be dealt in this paper. The aim of this study is to find out prototype of the residential house with ideal proportions, by using the rules of the Golden section and the law of Fibonacci sequence, and then examine advantages and disadvantages compared to the conventionally designed house. The idea is that every of designed elements is directly related to the golden ratio, meaning that overall dimensions of the plan, dimensions of each room individually and facades have the geometry that corresponds to geometry of golden rectangle. The work will be conceptualized on the specific model made in a computer program, Autodesk 3dsMax, and will be examined its functionality, aesthetics and modularity, which is very popular in contemporary practice. The methods used in this paper are analysis, synthesis, modal experiment and comparison.

Key words: Golden ratio, Fibonacci sequence, prototype of the residential house, proportions.

ON THE SEMIHOLONOMIC VELOCITIES AND CONTACT ELEMENTS

Miroslav Kures
Brno University of Technology, Czech Republic
kures@fme.vutbr.cz

Abstract

Semiholonomic jets and velocities play an important role in geometry and physics. The idea of semiholonomic velocities can be extended to the contact elements, [1]. We focus on algebraic and combinatorial properties of Weil algebras associated to higher order semiholonomic functors, [2].

References

[1] Kolar, I., Vitolo, R., *Absolute contact differentiation on submanifolds of Cartan space*, Differential Geometry and its Applications, Vol. 28 (2010), No. 1, pp. 19.

[2] Kures, M., *Weil algebras associated to functors of third order semiholonomic velocities,* Mathematical Journal of Okayama University, Vol.56, (2014), No.1, pp.117-127.

THE CONFORMAL MODELS OF FIBRATIONS DETERMINED BY THE ALGEBRA OF QUATERNIONS

Irina Kuzmina
Kazan Federal University, Russia
iranina@mail.ru

Abstract

Our aim is to study the principal bundles determined by the algebra of quaternions. The conformal model of the Hopf fibration is considered as example.

PSEUDO PARALLEL CR-SUBMANIFOLDS IN COMPLEX SPACE FORMS

Tee How Loo
Institute of Mathematical Sciences,
University of Malaya, 50603 Kuala Lumpur, Malaysia

looth@um.edu.my

Abstract

We classify pseudo parallel proper CR-submanifolds in non-flat complex space forms with CR-dimension greater than one. With this result, the non-existence of recurrent as well as semi parallel proper CR-submanifolds in non-flat complex space forms with CR-dimension greater than one can also be obtained.

Key words: complex space forms, CR-submanifolds, pseudo parallel submanifolds.

CERTAIN CURVATURE PROPERTIES OF GENERALIZED SASAKIAN-SPACE-FORMS

Pradip Majhi
Department of Mathematics,
University of North Bengal Raja Rammohunpur,
Darjeeling Pin-734013, West Bengal, India

mpradipmajhi@gmail.com

Abstract

The object of the present paper is to characterize generalized Sasakian-space-forms satisfying certain curvature restrictions. At first we consider ξ-conformally flat and ϕ-conformally flat generalized Sasakian-space-forms. Then we study ϕ-Weyl semi-symmetric generalized Sasakian-space-forms and obtain some interesting results which generalized some known results. Next we obtain a necessary and sufficient condition to be a generalized Sasakian-space-form is Weyl semi-symmetric. Finally, we construct some example to verify some results.

THE COMPONENTS OF THE STRUCTURE TENSOR OF ALMOST CONTACT MANIFOLDS WITH B-METRIC IN THE BASIC CLASSES AND THE SMALLEST DIMENSION

Hristo Manev[1] **and Mancho Manev**[2]
[1,2]Department of Algebra and Geometry, Faculty of Mathematics and Informatics,
Paisii Hilendarski University of Plovdiv, Bulgaria
[1]ico_manev@yahoo.com, [2]mmanev@uni-plovdiv.bg

Abstract

The present work deals with differential geometry of almost contact manifolds with B-metric. These manifolds are the odd-dimensional extension of almost complex manifolds with Norden metric and an almost neutral-signature analogue of almost contact manifolds with compatible metric. The differential geometry of the considered manifolds are developed but some important aspects are still obscure. The almost contact endomorphism ϕ is an almost complex structure on the contact distribution $ker(\eta)$ and ϕ acts as an anti-isometry with respect to the restriction of the B-metric g on $ker(\eta)$. In distinction from the Hermitian case, the basic B-metric generates an associated B-metric (not a Kähler form). Our main goal is to classify the almost contact manifolds with B-metric of dimension three - the smallest dimension of the manifold under consideration. We consider the structure tensor F generated by the covariant derivative of the almost contact endomorphism with respect to the Levi-Civita connection of the basic B-metric. There is known a classification regarding F made by G. Ganchev, V. Mihova and K. Gribachev in 1993. Following this classification, we determine the components of F in each of the eleven basic classes for the odd dimension in general. The case of the smallest dimension 3 is object of special interest in our consideration. We determine which of the eleven basic classes of the almost contact manifolds with B-metric are empty in the case of dimension 3. We adduce appropriate examples of the considered manifolds of dimension 3.

SASAKI-LIKE ALMOST CONTACT COMPLEX RIEMANNIAN MANIFOLDS

Mancho Manev[1], Stefan Ivanov[2] and Hristo Manev[3]
[1,3]Department of Algebra and Geometry, Faculty of Mathematics and Informatics,
Paisii Hilendarski University of Plovdiv, Bulgaria
[2] University of Sofia "St. Kliment Ohridski", Faculty of Mathematics and Informatics,
Blvd. James Baucher 5, 1164 Sofia, Bulgaria

[1]mmanev@uni-plovdiv.bg, [2]ivanovsp@fmi.uni-sofia.bg,
[3]ico_manev@yahoo.com

Abstract

A Sasaki-like almost contact complex Riemannian manifold is defined as an almost contact complex Riemannian manifold which complex cone is a holomorphic complex Riemannian manifold. Explicit compact and non-compact examples are given. A canonical construction producing a Sasaki-like almost contact complex Riemannian manifold from a holomorphic complex Riemannian manifold is presented and called an S^1-solvable extension.

ON SOME SPECIAL PARACONTACT METRIC MANIFOLDS

Veronica Martin-Molina
Centro Universitario de la Defensa de Zaragoza, Spain
vmartin@unizar.es

Abstract

The special class of paracontact metric (κ, μ)-spaces (M, ϕ, ξ, η, g) has the property $h^2 = (\kappa + 1)\phi^2$, which means that it is useful to distinguish three cases: $\kappa > -1$, $\kappa < -1$ and $\kappa = -1$. The fist two have been the object of study by several authors (see [1], for instance) but only some examples exist of the last one, which is possible only because the metric is not Riemannian.
We will endevour to provide a local classification of the paracontact metric $(-1, \mu)$-spaces in terms of the rank of h and we will present examples with every possible constant rank (see [2]).

References

[1] B. Cappelletti Montano, I. Küpeli Erken and C. Murathan, *Nullity conditions in paracontact geometry*, Differential Geom. Appl.,30 (2012) 665-693.

[2] V. Martin-Molina, *Paracontact metric manifolds without a contact metric counterpart.* arXiv:1312.6518v2.

SEMI-LOCAL LIOUVILLE EQUIVALENCE OF COMPLEX HAMILTONIAN SYSTEMS DEFINED BY RATIONAL HAMILTONIAN

Nikolay Nikolaevich Martynchuk
MSU, faculty of Mechanics and Mathematics, Russian Federation

mnick45@bk.ru

Abstract

Consider the complex Hamiltonian system defined by the Hamiltonian $f = z^2 + R(w)$, where z and w are two complex variables and R is a rational function. We say that f is a (hyperelliptic) rational Hamiltonian. Different rational Hamiltonians define, generally speaking, different Liouville foliations in the topological sense. In this talk we will present results related with the problem of classification of such foliations, especially - with the problem of semi-local topological classification.

Key words: Hamiltonian system, Liouville foliation.

References

[1] Kudryavtseva E.A., Lepskii T.A., *The topology of Lagrangian foliations of inegrable systems with hyperelliptic Hamiltonian.* Sb. Math. 2011. V.202. 3-4. P. 373-411.

CR-SUBMANIFOLDS WITH THE SYMMETRIC $\nabla\sigma$ IN A LOCALLY CONFORMAL KAEHLER SPACE FORM

Koji Matsumoto
2-3-65 Nishi-Odori, Yonezawa, Yamagata, 992-0059, Japan
tokiko_matsumoto@yahoo.com

Abstract

In this talk, we consider CR-submanifolds with the symmetric $\nabla\sigma$ which is a generalization of parallel second fundamental form, in a locally conformal Kaehler space form. Mainly, we prove that an l.c.K-space form is flat if its CR-submanifold has the symmetric $\nabla\sigma$ (See Theorem 3.1). Next, we consider the tensor field P defined in (1.4). And we show that, in an anti-holomorphic submanifold in an l.c.K.-space form, P is diagonal with respect to an adapted frame (See Theorem 3.2). Finally, we consider that, in a flat l.c.K.-space form, the Lee vector field satisfies a special properties (See Theorem 3.3).

References

[1] A. Bejancu, CR-submanifolds of a Kaehler manifold I, II, *Proc. Amer. Math. Soc.,* 69 (1978), 134 –142 and *Trans. Amer. Math. Soc.,* 250 (1979), 333–345.

[2] A. Bejancu, Geomerty of CR-submanifolds, *D. Reidel Publishing Company,* (1986).

[3] V. Bonanzinga and K. Matsumoto, Warped product CR-submanifolds in locally conformal Kaehler manifolds, *Periodica Math. Hungarica* 48 (2-2) (2004), 207–221.

[4] V. Bonanzinga and K. Matsumoto, On doubly warped product CR-submanifolds in a locally conformal Kaehler manifold,*Tensor (N.S.)* 69 (2008), 76–82.

[5] V. Bonanzinga and K. Matsumoto, Doubly warped product CR-submanifolds in a locally conformal Kaehler space form, *Acta Math. Acad. Paedagog. Nyházi. (N.S.)* 24 (2008), 93–102.

[6] B. Y. Chen, CR-submanifolds of a Kaehler manifold I and II, *J. of Differential Geometry,* 16, 305–322 and 493–509(1981).

[7] B. Y. Chen, Geometry of submanifolds, *New York, Marcel Dekker,* (1973).

[8] B. Y. Chen, Totally umbilical submanifolds, *Soochow J. Math.* **5** (1979), 9–37.

[9] S. Dragomir and L. Ornea, Locally Conformal K̈hler Geometry, *Birkhäuser*, (1998).

[10] T. Kashiwada, Some properties of locally conformal Kähler manifolds, *Hokkaido Math, J.*, **8** (1979), 191–198.

[11] K. Matsumoto, Locally conformal Kähler manifolds and their submanifolds, *Mem. Sec, Acad. Română. Ser. IV* **14** (1991), 7-49(1993).

[12] K. Matsumoto, On CR-submanifolds of locally conformal Kähler manifolds I, II, *J. Korean Math.* **21** (1984), 49–61 and *Tensor N, S.,* **45** (1987), 144–150.

[13] K. Matsumoto, Twisted product CR-submanifolds in a locally conformal Kaehler manifold, preprint.

[14] K. Matsumoto and Z. Şentürk, Certain twisted product CR-submanifolds in a Kaehler manifold, *Proc. of the conference RIGA 2011, Riemannian Geometry and Applications*, 2011.

[15] A. Mihai, Modern Topics in Submanifold theory, *University of Bucharest,* (2006).

[16] B. O'Neil, Semi-Riemannian Geometry with Applications to Relativity, *New York: Academic Press,* (1983).

[17] I. Vaisman, Locally conformal almost Kähler manifolds, *Israel J, Math.*, **24**(1976), 338–351.

[18] K. Yano, Differential geometry on complex and almost complex spaces, *Pergamon Press,* (1965).

ON A VARIATIONAL PROBLEM WITH FREE BOUNDARY ON TWO PARALLEL PLANES

Monica Merkle
Universidade Federal do Rio de Janeiro, Brasil
monica@im.ufrj.br

Abstract

We present some results on the geometry of surfaces with mean curvature that is linear on its height over a flat plane and has boundary on two parallel planes.

ON SUBMANIFOLDS OF STATISTICAL MANIFOLDS

Adela MIHAI[1], **M. Evren AYDIN**[2] **and Ion MIHAI**[3]
[1]Department of Mathematics and Computer Science,
Technical University of Civil Engineering Bucharest, Romania
and
Department of Mathematics, Faculty of Mathematics and Computer Science,
University of Bucharest, Bucharest, Romania
[2]Department of Mathematics, Faculty of Science, Firat University, Elazig, Turkey
[3]Department of Mathematics, Faculty of Mathematics and Computer Science,
University of Bucharest, Romania

[1]adela_mihai@fmi.unibuc.ro

Abstract

The geometry of statistical manifolds (Amari [1]) deals with dual connections, also called conjugate connections in affine geometry, being therefore closely related to affine differential geometry. Also, a statistical structure is a generalization of a Hessian structure (Shima [2]).

In the geometry of submanifolds, Gauss and Weingarten formulae and the equations of Gauss, Codazzi and Ricci are the fundamental equations. Corresponding fundamental equations on statistical submanifolds were given by Vos [3].

We began to study the behaviour of submanifolds in statistical manifolds of constant curvatures. We investigate curvature properties of such submanifolds.

AMS Subject Classification: 53C05, 53C40, 53A40.

Key words: Hessian manifold, statistical manifold, dual connections, submanifolds.

References

[1] S. Amari, *Differential-Geometrical Methods in Statistics*, Springer-Verlag, 1985.

[2] H. Shima, *The Geometry of Hessian Structures*, World Scientific Publ., 2007.

[3] P. W. Vos, *Fundamental equations for statistical submanifolds with applications to the Bartlett correction*, Ann. Inst. Statist. Math. **41(3)** (1989), 429-450.

ON GENERALIZED WINTGEN INEQUALITY

Ion Mihai
University of Bucharest, Romania
imihai@fmi.unibuc.ro

Abstract

The normal scalar curvature conjecture, also known as the DDVV conjecture, was formulated by De Smet et al. (1999). It was proven recently by Lu (2011) and by Ge and Tang (2008) independently.
We established the DDVV inequality, also known as the generalized Wintgen inequality, for Lagrangian submanifolds in complex space forms (2014). Some applications were given. Also an inequality for slant submanifolds in complex space forms is stated.
We obtain a corresponding inequality for Legendrian submanifolds in Sasakian space forms and derive some applications.

GEODESIC AND HOLOMORPHICALLY PROJECTIVE MAPPINGS

Josef Mikeš[1] **and Irena Hinterleitner**[2]
[1]Palacky University Olomouc, Czech Republic
[2]Brno University of Technology, Czech Republic

[1]josef.mikes@upol.cz, [2]hinterleitner.irena@seznam.cz

Abstract

The lecture is devoted to the geodesic and holomorphically projective mapping theory of (pseudo-) Riemannian manifolds with respect to differentiability of their metrics. Most of the results in this area is formulated for "sufficiently" smooth, or analytic, geometric objects, as usual in differential geometry. We may find it in most monographs and research, dedicated to the study of theory geodesic and holomorphically projective mappings and transformations.

CALASSICAL LINEAR CONNECTIONS FROM PROJECTABLE ONES ON VERTICAL WEIL BUNDLES

Włodzimierz Mikulski[1] and Jan Kurek[2]
[1]Institute of Mathematics, Jagiellonian University, Cracow, Poland
[2]Institute of Mathematics, Maria Curie-Sklodowska University of Lublin, Poland
[1]wjelon@pk.edu.pl, [2]kurek@hektor.umcs.lublin.pl

Abstract

In [1], given a Weil algebra A, $k+1 = \dim_{\mathbb{R}} A$, we reduced the problem of finding of all natural operators $\Lambda\colon Q \rightsquigarrow QT^A$ lifting classical linear connections to the Weil functor T^A to the one of finding of all natural operators $C\colon Q \rightsquigarrow (T \times \ldots \times T, T^* \otimes T^* \otimes T)$ sending classical linear connections $\nabla \in Con(M)$ into fibred maps $C_M(\nabla)\colon TM \times_M \ldots \times_M TM$ (k times of TM) $\to T^*M \otimes T^*M \otimes TM$. In the present note we study the same problem for T^A replaced by the vertical Weil functor V^A. More precisely, we reduce the problem of finding of all natural operators $\Lambda\colon Q_{proj} \rightsquigarrow QV^A$ lifting projectable classical linear connections to V^A to the one of finding of all natural operators $C\colon Q_{proj} \rightsquigarrow (V \times \ldots \times V, F_1^* \otimes F_2^* \otimes F_3)$ sending fibred maps $C_Y(\nabla)\colon VY \times_Y \ldots \times_Y VY$($k$ times of VY) $\to (F_1Y)^* \otimes (F_2Y)^* \otimes F_3Y$ covering id_Y, where F_1, F_2, F_3 are T or V, and where T is tangent functor and V is the vertical functor.

References

[1] J. Kurek and W. M. Mikulski, *On lifting connections to Weil bundles*, Ann. Polon. Math. 103(3)(2012), 319-324.

COHOMOLOGY OF NILPOTENT LIE ALGEBRAS

Dmitry Millionshchikov
Moscow State University, Russia
mitia_m@hotmail.com

Abstract

We will discuss computation of the cohomology of infinite and finite dimensional nilpotent Lie algebras together with applications to some geometrical and topological problems.

TEMPORARY CHANGES IN GEOMETRY OF MEMBRANE STRUCTURES CAUSED BY LIVE LOADS

Vuk Milošević
Faculty of Civil Engineering and Architecture, University of Niš, Serbia
vukamer@yahoo.com

Abstract

Membrane structures are one of the newest and the most attractive structural systems in the world. Their doubly curved form with negative Gaussian curvature is much more than just an esthetical element. It plays an important structural role in ensuring the stability of the membrane. Due to the elasticity and low stiffness of the membrane, its geometry will suffer temporary changes under the influence of external loading. This change can be of great importance for the structural calculation of the membrane.

QC HYPERSURFACES IN HYPER-KAEHLER GEOMETRIES

Ivan Minchev
University of Sofia "St. Kliment Ohridski", Faculty of Mathematics and Informatics,
Blvd. James Baucher 5, 1164 Sofia, Bulgaria

minchevim@yahoo.com

Abstract

In my talk I will present a method for studying quaternionic contact (qc) hypersurfaces in hyper-Kaehler manifolds. This method, based on a certain volume normalization of the intrinsic qc structure, allows us to describe completely the set of all qc hypersurfaces in the flat quaternionic vector space. More generally, we will show that if a hyper-Kaehler manifold admits a qc hypersurface, then the Riemannian curvature tensor needs to be degenerate along this hypersurface.

ON SOME PROPERTIES OF NON-SYMMETRIC CONNECTIONS

Svetislav Minčić
Faculty of Science and Mathematics, University of Niš, Serbia
svetislavmincic@yahoo.com

Abstract

We consider a non-symmetric connection on a differentiable manifold. On the basis of the non-symmetry, there are possible four kinds of covariant derivatives and, in relation with that, some properties of corresponding spaces are considered.

SEMI-HAMILTONIAN SYSTEM FOR INTEGRABLE GEODESIC OWS ON 2-TORUS

Andrey Mironov
Moscow State University, Russia
mironov@math.nsc.ru

Abstract

We prove that the question of existence of polynomial rst integrals of the geodesic ow on 2-torus leads to a semi-Hamiltonian quasi-linear equations, i.e. the system can be written in the conservation lows form and in the hyperbolic region it has Riemannian invariants. We also prove that in the elliptic region cubic and quartic integrals are reduced to the integrals of degree one or two. The results obtained with M.Bialy.

OPTIMAL PACKINGS BY TRANSLATION BALLS IN THE $\widetilde{\mathbf{SL_2R}}$ GEOMETRY

(joint work with Jenő SZIRMAI (Budapest) and Andrei Yu. VESNIN (Novosibirsk))

Emil Molnár
Budapest University of Technology and Economics Institute of Mathematics
and Economics Institute of Mathematics
Department of Geometry, Budapest, P. O. Box: 91, H-1521, Hungary

emolnar@math.bme.hu

Abstract

$\widetilde{\mathbf{SL_2R}}$, i.e. the universal cover of the 2×2 real matrices of unit determinant, is one of the homogeneous 3-dimensional Riemannian geometries, the 8 THURSTON spaces: $\mathbf{E^3}, \mathbf{S^3}, \mathbf{H^3}$, $\mathbf{S^2} \times \mathbf{R}, \mathbf{H^2} \times \mathbf{R}$, $\widetilde{\mathbf{SL_2R}}$, **Nil** and **Sol**. Classical problems of the Euclidean geometry $\mathbf{E^3}$, or of the spherical ($\mathbf{S^3}$) and hyperbolic ($\mathbf{H^3}$) spaces, can be asked also in $\widetilde{\mathbf{SL_2R}}$. The most analogous geometry to $\widetilde{\mathbf{SL_2R}}$ is the BOLYAI-LOBACHEVSKY hyperbolic space $\mathbf{H^3}$ and the direct product $\mathbf{H^2} \times \mathbf{R}$, since their plane geometries are the same. The one-sheeted hyperboloid solid models $\widetilde{\mathbf{SL_2R}}$ in the projective spherical space $\mathcal{PS}^\mathbf{3}$ by the real vector space $\mathbf{V^4}$ and its dual by the usual subspace structure. Thus the classical ball packing problem can be attacked in $\widetilde{\mathbf{SL_2R}}$ as well, in the sense of Johannes Kepler, but some strange phenomena occur. Our method is based on the intensive use of computer procedures, developed in recent works of the authors.

References

[1] Molnár, E., *Polyhedron complexes with simply transitive group actions and their realizations,* Acta Math. Hung., **38**, 175-216 (1997).

[2] Molnár, E., *The projective interpretation of the eight 3-dimensional homogeneous geometries,* Beitr. Algebra Geom, **38**(2), 261-288 (1997).

[3] Molnár, E., Szilágyi, B., *Translation curves and their spheres in homogeneous geometries,* Publ. Math. Debrecen, **78**, 327-346 (2011).

[4] Molnár, E., Szirmai, J., *Symmetries in the 8 homogeneous 3-geometries,* Symmetry Cult. Sci., **21**, 87-117 (2010).

[5] Molnár, E., Szirmai, J., Vesnin, A., *Projective metric realizations of cone-manifolds with singularities along 2-bridge knots and links,* J. Geom. **95**, 91-133 (2009).

[6] Molnár, E., Szirmai, J., Vesnin, A., *Packings by translation balls in* $\widetilde{\mathbf{SL_2R}}$, to appear in J. Geometry (2014).

AXIOMATIC CHARACTERIZATION OF QUASI-UNIFORM SPACES AND SOME OF ITS APPLICATIONS

M.N. Mukherjee
Department of Pure Mathematics University of Calcutta 35,
Ballygunge Circular Road Kolkata- 700019, West Bengal, India

mukherjeemn@yahoo.co.in

Abstract

There is known characterization of uniform spaces in terms of uniform covers. But quasi uniform covers fail to characterize quasi-uniform spaces, even if the quasi-uniformity of the space is transitive. The intent of the first part of the present deliberation is to demonstrate that under suitable modification of the definition of quasi-uniform cover, one may crack the open problem and achieve the desired axiomatic formulation of the quasi-uniformity of a quasi-uniform space. Moreover, it is well known from a classical paper of Pervin that the topology of any arbitrary topological space is induced by a quasi uniformity on the ambient set. As a consequence, it can well be conjectured that different topological invariants can then be formulated in terms of the said type of covers. That this is actually so, is demonstrated in the second part of the deliberations.

AMS Subject Classification: 54E15, 54E99.

Key words: Quasi-uniformity, transitive base (subbase), strong quasi-uniform cover.

ON SOME PERIODIC MAGNETIC CURVES

Marian Ioan Munteanu
Alexandru Ioan Cuza University of Iasi, Romania
marian_ioan_munteanu@yahoo.com

Abstract

It is an interesting question whether a given equation of motion has a periodic solution or not, and in the positive case to describe them. We investigate periodic magnetic curves in elliptic Sasakian space forms and we obtain a quantization principle for periodic magnetic flowlines on Berger spheres. We give a criterion for periodicity of magnetic curves on the unit sphere S^3.

References

[1] J. Inoguchi, M.I. Munteanu, *Periodic magnetic curves in elliptic Sasakian space forms*, arXiv:1310.2899 [math.DG].

ON LOCALLY CONFORMAL KAEHLER SPACE FORMS

Pegah MUTLU[1] **and Zerrin ŞENTÜRK**[2]
[1,2]Istanbul Technical University, Faculty of Science and Letters,
Mathematics Engineering Department, Maslak, TR-34469, Istanbul, Turkey
[1]sariaslani@itu.edu.tr, [2]senturk@itu.edu.tr

Abstract

The notion of a locally conformal Kaehler manifold (an l.c.K-manifold) in a Hermitian manifold has been introduced by I. Vaisman in 1976. In this work, we study some properties of l.c.K-space form with the tensor P_{ij} is not hybrid. Moreover, the Sato's form of the holomorphic curvature tensor in almost Hermitian manifolds and l.c.K-manifolds are presented.

AMS Subject Classification: 53C40.

Key words: Locally conformal Kaehler manifold, Lee form, Locally conformal Kaehler space form, hybrid, holomorphic curvature tensor.

References

[1] Kashiwada, T., *Some Properties of Locally Conformal Kaehler Manifolds,* Hokkaido Mathematical Journal Vol.8 1978.

[2] Matsumoto, K., *Locally Conformal Kaehler Manifolds And Their Submanifolds,* MEMORIILE SECŢIILOR ŞTIINŢIFICE, XIV 1991.

[3] Prvanovic, M., *On a curvature tensor of Kaehler type in an almost Hermitian and almost para-Hermitian manifold,* Mat. Vesnik, 50, No. 1-2 1998.

[4] Prvanovic, M., *Some Properties of the Locally Conformal Kaehler Manifold,* SRC PBull. Cl. Sci. Math. Nat. Sci. Math. No. 35 2010.

ORBITAL EQUIVALENCE OF SOME CLASSICAL INTEGRABLE SYSTEMS

Stanislav Nikolaenko
Moscow State University, Russia

nikostas@mail.ru

Abstract

We discuss the problem of the existence of the orbital isomorphisms between several classical integrable Hamiltonian systems (see [1], [2], [3]). By an orbital isomorphism we mean a homeomorphism (diffeomorphism) mapping the oriented trajectories of one system to those of the other system. For establishing the orbital equivalence (or non-equivalence) of some systems, the Bolsinov-Fomenko invariant ("t-molecule") [1], [4] is used. In particular, we show the orbital equivalence of the three well-known integrable systems: the Euler case in rigid body dynamics, the Jacobi problem about geodesics on the ellipsoid and the Chaplygin case in dynamics of a rigid body in fluid.

References

[1] A. V. Bolsinov and A. T. Fomenko. *Trajectory invariants of integrable Hamiltonian systems. The case of simple systems. Trajectory classification of Euler-type systems in rigid body dynamics.* Izvest. Akad. Nauk SSSR, Ser. Matem., 59 (1995), No. 1, P. 65-102 (in Russian).

[2] A. V. Bolsinov and A. T. Fomenko. *Orbital classification of the geodesic flows on two-dimensional ellipsoids. The Jacobi problem is orbitally equivalent to the integrable Euler case in rigid body dynamics.* Funkts. Analiz i ego Prilozh., 29 (1995), No. 3, P. 1-15 (in Russian).

[3] A. T. Fomenko and S. S. Nikolaenko. *The Chaplygin case in dynamics of rigid body in fluid is orbitally equivalent to the Euler case in rigid body dynamics and to the Jacobi problem on geodesics on the ellipsoid.* J. Geometry and Physics (to appear).

[4] A. V. Bolsinov and A. T. Fomenko. *Orbital equivalence of integrable Hamiltonian systems with two degrees of freedom. A classification theorem. I, II.* Matem. Sbornik, 185 (1994), No. 4, P. 27-80; No. 5, P. 27-78 (in Russian).

POSSIBILITIES OF APPLICATION OF CONES WITH A TORUS KNOT AS DIRECTRIX AND VERTEX ON IT IN THE FORM OF ARCHITECTURAL STRUCTURES

Vladan Nikolić
The Faculty of Civil Engineering and Architecture, University of Niš, Serbia
vladan_nikolic@yahoo.com

Abstract

A Torus knot (p, q) is obtained by looping a string through the hole of a torus p times with q revolutions before joining its ends, where p and q are relatively prime. A Torus knot (p, q) is a space curve of some order n. Each torus knot can be used as a directrix for a derivation of some surface. A Torus knot (p, q) of the n-th order can be connected with one particular point in space, in order to produce a n-th order surface, a cone. If that point (the vertex of the cone) lies on that torus knot, the directrix of the cone, then the bisectrices of that directrix form the cone of $(n-1)$-th order.

By using segments and cuttings of those cones, it is possible to form complex, attractive and rational spatial structures in architecture. Given that those surfaces are ruled, developable, single curved surfaces, a relatively simple practical construction from the spatial structures created in this way is possible.

The paper considers the possibilities of applications of those cones with aspects of geometric construction, aesthetic qualities and rationality of resulting spatial structures. The figure shows the torus knot $(1, 1)$, as well as an architectural object based on the conical surface shown in version number 5 of the same figure.

MAGNETIC CURVES IN QUASI-SASAKIAN MANIFOLDS

Ana Irina Nistor
Gheorghe Asachi Technical University of Iasi, Romania
ana.irina.nistor@gmail.com

Abstract

A product of a $(2p+1)$-dimensional Sasakian and a $(2k)$-dimensional Kaeher manifold carries a natural quasi-Sasakian structure.

We investigate the magnetic curves corresponding to the magnetic field defined by the fundamental 2-form and we prove that they have maximum order 5.

We sustain our result with some examples providing also explicit parametrizations for the magnetic curves.

ON THE CURVATURE OF NULL CURVES IN LORENTZIAN 3-SPACES

Zbigniew Olszak
Institute of Mathematics and Computer Sciences,
Wroclaw University of Technology, Poland

zbigniew.olszak@pwr.wroc.pl

Abstract

For null curves in Lorentzian-Minkowski 3-spaces, certain relations between the Cartan curvatures and the equiaffine curvatures and the Schwarzian derivatives of some functions related to the curves will be presented.

GEOMETRY OF NULL HYPERSURFACES

Oscar Palmas
Universidad Nacional Autonoma de Mexico, Mexico
oscar.palmas@ciencias.unam.mx

Abstract

First we study the null hypersurfaces of the Minkowski space, classifying all of their rotation null hypersurfaces. Then we start our analysis of the submanifold geometry of the null hypersurfaces. In the particular case of the $(n+1)$-dimensional lightcone, we characterize its totally umbilical spacelike hypersurfaces, show the existence of non-totally umbilical ones and give a uniqueness result for the non-totally umbilical, minimal rotation surfaces in the 3-dimensional lightcone.

AMS Subject Classification: 34A09, 37C10, 53A10, 53B30.

Key words: Null hypersurfaces, totally umbilical hypersurfaces.

CURVATURE CONDITIONS ON $\delta(2,2)$ IDEAL SUBMANIFOLDS

Anica Pantić
Faculty of Science, University of Kragujevac, Serbia
anica.pantic@kg.ac.rs

Abstract

The submanifolds M^n of E^{n+m} for which B.-Y. Chen's [1] inequality

$$\delta(2,2,\ldots,2) \leq \{n^2[(n-k)-1]/[2(n-k)]\}.H^2$$

at all of their points is an equality, are called $\delta(2,2,\ldots,2)$ *Chen ideal submanifolds.* For $\delta(2,2,\ldots,2)$ Chen ideal submanifolds it is shown [2] that the (intrinsic) Ricci principal directions and the (extrinsic) Casorati principal directions coincided. In this paper we investigate some curvature conditions on such submanifolds.

References

[1] B.-Y. Chen, *Pseudo-Riemannian Geometry, δ-invariants and Applications*, World Scientific, Hackensack, New Jersey, (2011).

[2] S. Decu, A. Pantić, M. Petrović-Torgašev and L. Verstraelen, *Ricci and Casorati principal directions of $\delta(2)$ Chen ideal submanifolds*, Kragujevac J. Math., Vol 37 No 1 (2013), 25-31.

THE SHARP LOWER BOUND OF THE FIRST EIGENVALUE OF THE SUB-LAPLACIAN ON A QUATERNIONIC CONTACT MANIFOLD

Alexander Petkov
University of Sofia, Faculty of Mathematics and Informatics, Bulgaria
a_petkov_fmi@abv.bg

Abstract

In this talk we give analogues of the classical theorem of Lichnerowicz in the case of quaternionic contact manifolds. The Lichnerowicz-type theorem says, that under some condition imposed on the Ricci tensor and the torsion tensor of the Biquard connection of a compact quaternionic contact manifold, it is obtained a sharp lower bound of the first eigenvalue of the sub-Laplacian. The estimate is sharp in the sense, that the equality in the corresponding inequality is obtained on the 3-Sasakian sphere. To obtain the estimate on the seven-dimensional quaternionic contact manifolds, we need by an exstra assumption, concerning the non-negativity of the P-function of any eigenfunction.

TOPOLOGICAL K-THEORY AND SPACES OF MATRICES

Zoran Petrović
Faculty of Mathematics, University of Belgrade, Serbia
zoranp@matf.bg.ac.rs

Abstract

A problem of determining maximal dimension of spaces of matrices satisfying some conditions on their rank is an old problem. We show how one can use topological K-theory to get some results in that area.

BOCHNER-FLAT KÄHLER MANIFOLD AND THE COMPATIBILITY OF RICCI TENSOR

Mileva Prvanović
Mathematical Institute, SANU, Belgrade, Serbia

Abstract

The Ricci tensor of a conformally flat Riemannian manifold is Riemannian compatible, that is it satisfies the condition

$$R_{ia}R^{a}_{hjk} + R_{ja}R^{a}_{hki} + R_{ka}R^{a}_{hij} = 0. \tag{1}$$

It is not the same for the Kähler manifolds: in general, Ricci tensor of a Bochner-flat Kähler manifold does not satisfy the condition (1).

The aim of the present communication is to determine the necessary and the sufficient conditiohn for such compatibility.

ORNAMENTS OF SERBIAN MEDIEVAL FRESCOES

Ljiljana Radović[1] **and Slavik Jablan**[2]
[1]Faculty of Mechanical Engineering, University of Niš, Serbia
[2]Mathematical Institute Belgrade, Serbia
[1]ljradovic@gmail.com, [2]slavik.jablan@ict.edu.rs

Abstract

During the complete historical development of humanity, there existed unbreakable connections between geometry and art, where the visual presentation often served as the basis for geometrical consideration. The first significant incitement for the study of ornamental art came from mathematicians.The approach to the classification and analysis of ornaments based on symmetries was enriched by the contributions of different authors (A. Müller, A.O. Shepard, N.V. Belov, D. Washburn, D. Crowe, B. Grünbaum...). In this work we analyse and classify ornaments of Serbian Medieval Frescoes.

Key words: Ornaments, symmetry groups, color symmetry.

CMC CONSTANT ANGLE SURFACES IS SPACE FORMS

Gabriel Ruiz[1], **Cinthia Barrera**[2] **and Antonio Di Scala**[3]
[1,2]Institute of Mathematics National University of Mexico, Mexico
[3]Torino, Italy

[1]gruiz@matem.unam.mx, [2]aihtnic@matem.unam.mx
[3]antonio.discala@polito.it

Abstract

We prove a generalization of the Laplacian of a support function of a hypersurface. This allows us to study the constant mean curvature surfaces in space forms which have constant angle with respect to a closed and conformal vector field. The result we find says that these surfaces are totally umbilic.

SOME WARPED PRODUCT SUBMANIFOLDS OF A KENMOTSU MANIFOLD

Mohammad Shuaib
Department of Mathematics, Aligarh Muslim University, Aligarh 202 002, India
Shuaibyousuf6@gmail.com

Abstract

Many differential geometric properties of a submanifold of a Kaehler manifold are conceived via canonical structure tensors T and F on the submanifold. For instance, a CR-submanifold of a Kaehler manifolds a CR-product if and only if T is parallel on the submanifold. Warped product submanifolds are the generalized version of CR-product submanifolds. Therefore, it is natural to see how the non-triviality of the covariant derivatives of T and F gives rise to warped product submanifolds. In the present article, we have worked out characterizations in terms of T and F under which a contact CR-submanifold of a Kenmotsu manifold reduces to a warped product submanifold.

SOME PROPERTIES OF ET-PROJECTIVE TENSORS OBTAINED BY WEYL PROJECTIVE TENSOR

Mića S. Stanković[1], **Milan Lj. Zlatanović**[2], **Nenad O. Vesić**[3]
[1,2,3] Faculty of Science and Mathematics, University of Niš, Serbia
[1]stmica@ptt.rs,[2]zlatmilan@pmf.ni.ac.rs,[3]vesic.specijalac@gmail.com

Abstract

Vanishing of curvature tensors and projective curvature tensors of a non-symmetric affine connection space as functions of vanished curvature tensor of the associated space are analyzed in the first part of this paper. In the second part of it it is analyzed projective curvature tensors of a non-symmetric affine connection space as a linear function of the Weyl projective tensor of the associated space of this space.

AMS Subject Classification: 53B05.

Key words: geodesic mapping, invariant, curvature tensor, projective curvature tensor, flatness.

HODGE - DE RHAM AND TACHIBANA OPERATORS ON COMPACT RIEMANNIAN MANIFOLDS WITH THE BOUNDED POSITIVE AND NEGATIVE CURVATURE OPERATOR OF THE SECOND KIND

Sergey Stepanov[1], Irina Tsyganok[2] and Josef Mikeš[3]
[1,2]Finance University under the Government of Russian Federation
[3]Palacky University Olomouc, Czech Republic
[1]s.e.stepanov@mail.ru,[2]i.i.tsyganok@mail.ru
[3]josef.mikes@upol.cz

Abstract

Spectral geometry is the field of mathematics which concerns relationships between geometric structures of an n-dimensional Riemannian manifold (M, g) and the spectra of canonical differential operators. In the present report we show spectral properties of a little-known natural Riemannian second-order differential operator acting on differential forms.

In the first section of our report we consider the basis of the space of natural (with respect to isometric diffeomorphisms of Riemannian manifolds) first-order differential operators on differential r-forms ($2 < r < n-2$) with values in the space of homogeneous tensors on (M, g). We show that this basis consists of three following operators d, d^*, D where d is the exterior differential, d^* is the formal adjoint to d exterior codifferential and D is a conformal differential [1].

Next, using basis operators, we construct the well-known the Hodge-de Rham Laplacian $\Delta = dd^* + d^*d$ and the little-known Tachibana operator $\square := r(r+1)D^*D$. We recall that the Hodge-de Rham operator D is natural, elliptic, self-adjoint second order differential operator acting on differential forms on (M, g). If (M, g) is compact, spectrums of such operator is an infinite divergent sequence of real numbers, each eigenvalue being repeated according to its finite multiplicity [2, p. 273-321]. In addition, we find the lower bound of the first eigenvalue of D on a compact (M, g) with positive curvature operator of the second kind [3].

In the final part of our report we recall that the Tachibana operator $\square$ is a natural, elliptic, self-adjoint second order differential operator acting on smooth differential forms on (M, g), too [4]. In addition, we prove that $\square$ has positive eigenvalues, the eigenspaces of $\square$ are finite dimensional and eigenforms corresponding to distinct eigenvalues are orthogonal. In addition, we find the lower bound of the first eigenvalue

of $\square$ on a compact (M, g) with positive curvature operator of the second kind.

Key words: Riemannian manifold, natural elliptic second order linear differential operators on differential forms, eigenvalues, eigenforms.

References

[1] Stepanov S.E., *A new strong Laplacian on differential forms,* Mathematical Notes 76(2004), 420-425.

[2] Craioveanu M., Puta M., Rassias T. M.,*Old and new aspects in spectral geometry,* Kluwer Academic Publishers, London, 2001.

[3] Bouguignon J.-P., Karcher H., *Curvature operators: pinching estimates and geometric examples,* Ann. Sc. Éc. Norm. Sup. 11(1978), 71-92.

[4] Mikeš J., Stepanov S.E., *Betti and Tachibana numbers of compact Riemannian manifolds,* Differential Geometry and its Applications 31(2013), 486-495.

THE FIELD OF LINEAR ENDOMORPHISMS ATTACHED TO A GEODESIC MAPPING

E.S. Stepanova[1] **and Josef Mikeš**[2]
[1,2]Depart. of Algebra and Geometry,
Palacky University, Olomouc, Czech Republic
[1]stephelena@list.ru, [2]josef.mikes@upol.cz

Abstract

Let (M, g) and (M', g') be two Riemannian manifolds of equal dimensions. The diffeomorphism $f\colon (M, g) \to (M', g')$ is called a *geodesic mapping* [1, p. 127] if it maps all geodesic curves of (M, g) onto geodesic curves in (M', g'). Moreover, if this mapping preserves the natural parameters of geodesic curves, then f is called *affine*.

It is well-known [1, p. 167] that for any geodesic mapping $f\colon (M, g) \to (M', g')$ we can define a tensor field A_f of type (1,1), namely, a field of linear endomorphisms of the tangent space T_xM at each point x of M. If λ_1, λ_2 are distinct eigenvalue functions of A_f, defined in a connected component of $M_f \subset M$ and a geometric multiplicity of λ_1 is at least two, then the eigenspace distribution D_1 is integrable and each maximal integral manifold of D_1 is a totally geodesic submanifold of (M, g); λ_1 is constant along D_1 and $traceA_f$ is constant along D_2. In addition, if geometric multiplicity of every eigenvalue function of A_f, defined in a connected component of $M_f \subset M$, is at least two, then $traceA_f$ is constant along M_f and the mapping $f\colon (M, g) \to (M', g')$ is affine. Moreover, if the field of linear symmetric endomorphisms A_f has n distinct eigenvalues at all points of a dense subset of (M, g), then, for any point x of M, the curvature tensor R_x is diagonalised by some orthonormal basis of the tangent space T_xM, diagonalizing $A_{f(x)}$. Moreover, in this case the Pontryagin forms of (M, g) and the real Pontryagin classes of (M, g) are all zero [2].

AMS Subject Classification: 53C20, 53C21, 53C22, 53C23.

Key words: Riemannian manifold, geodesic mapping, eigenvalue function of the tensor field.

References

[1] Mikeš, J., Vanzurova A., Hinterleitner I., *Geodesic mappings and some generalizations,* Olomouc, Palacky University Press, 2009.

[2] Milnor J.W., Stasheff J.D., *Characteristic classes,* Princeton University Press, Princeton, 1974.

SUPERGROUPS OF HYPERBOLIC SPACE GROUPS WITH SIMPLICIAL DOMAINS

Milica Stojanović
Faculty of Organizational Sciences, University of Belgrade, Serbia
milicas@fon.rs

Abstract

Hyperbolic space groups are isometry groups, acting discontinuously on the hyperbolic 3-space with compact fundamental domain. One possibility to classify them is to look for fundamental domains of these groups. In the papers of E. Molnár, I. Prok, J. Szirmai groups with simplicial and half-simplicial domains are classified into 32 families. Cases of not maximal groups appear in the first 12 families.
In the previous investigation of supergroups such not maximal groups there are considered all of them except these belonging to families $F2$ and $F5$. Here are given results for series of groups in these two families.

COLORED HOMFLY HOMOLOGY OF KNOTS AND LINKS AND GENERALIZED VOLUME CONJECTURE

Marko Stošić
Mathematical Institute, SANU, Belgrade, Serbia

mstosic@isr.ist.utl.pt

Abstract

In this talk I will present recent advances on the understanding of the colored HOMFLY homology of knots and links, by using different approaches from mathematics and physics. We describe the structural properties of these homologies and show how these can be used in the generalized volume conjecture that relates the asymptotic behaviour of the colored HOMFLY homology and new invariant - so-called quantum super-A-polynomial, generalizing the classical A-polynomial of a knot.

GEOMETRY OF 4-DIMENSIONAL NILPOTENT LIE GROUPS WITH NEUTRAL SIGNATURE

T. Šukilović
University of Belgrade, Faculty of Mathematics, Belgrade, Serbia
tijana@matf.bg.ac.rs

Abstract

In 1986. Magnin proved that, up to isomorphism, there are only two non-Abelian nilpotent Lie algebras of dimension 4 : $\mathfrak{h}_3 \oplus \mathbb{R}$ and $\mathfrak{g}_4$ with corresponding Lie groups $H_3 \times \mathbb{R}$ and G_4.

Classification of Riemannian manifolds in dimension 4 has been done by Lauret, while Guediri showed that on a 2-step nilpotent Lie group all left invariant pseudo-Riemannian metrics are geodesically complete. In the same paper, he found an example of a non-complete left invariant Lorentz metric on a 3-step nilpotent Lie group that corresponds to Lie algebra $\mathfrak{g}_4$.

In the present work we classify, up to automorphism, left invariant metrics with $(2, 2)$ signature on 4-dimensional nilpotent Lie groups. We investigate their geometry, especially holonomy groups and decomposability of these metrics.

The structure of isometry group of nilpotent group with left invariant metric in pseudo-Riemannian setting had been presented by Cordero and Parker. Also, Hall and Wang recently investigated projective structure in 4-dimensional manifolds with metric of neutral signature. Briefly, we recall on their results.

Finally, using an algorithm proposed by Matveev, we find projective classes of the metrics and prove all of these metrics are also left invariant.

In addition, we compare obtained results with cases of Riemannian and Lorentz metrics.

AMS Subject Classification: 53A06, 22E06.

Key words: nilpotent group, holonomy group, geodesically equivalent metrics.

ON THE LINEARLY INDEPENDENT VECTOR FIELDS ON GRASSMANN MANIFOLDS

Kostadin Trenčevski
Institute of Mathematics, Ss. Cyril and Methodius University,
Arhimedova 3, P.O.Box 162, 1000 Skopje, Macedonia

kostatre@pmf.ukim.mk

Abstract

In this paper are found $\theta(n)$ linearly independent vector fields on the Grassmann manifold $G_k(V)$ of k-planes in n-dimensional Euclidean vector space if k is odd number, where $\theta(n)$ is the maximal number of linearly independent vector fields on S^{n-1}, i.e. skewsymmetric anti-commuting complex structures on $\mathbb{R}^n$.

AMS Subject Classification: 53C30.

Key words: Grassmann manifold, vector field, tangent space, complex structure.

TITLE MINIMAL SPANNING TREES ON INFINITE SETS

Alexey Avgustinovich Tuzhilin
Lomonosov Moscow State University Faculty of Mechanics and Mathematics, Chair of Differential Geometry and Applications, Russian Federation
tuz@mech.math.msu.su

Abstract

In this talk, we discuss minimal spanning trees on metric spaces consisting of infinitely many points. The finite case is well studied and has many applications. The infinite case may be applied to investigation of the Steiner ratio (for example, in the case of Euclidean 3-dim space the best known estimate of the Steiner ratio is achieved as a limit of some sequence of finite boundary sets whose cardinality tends to infinity). Previously, A.O. Ivanov, I.M. Nikonov, and A.A Tuzhilin obtained complete description of those metric spaces, which can be spanned by trees of finite length. Now we present some conditions guaranteed that for such sets there exist minimal spanning trees.

AMS Subject Classification: 51F99.

Key words: Minimal spanning tree.

LIE ALGEBRAS OF PURE BRAID GROUPS AND PURE MAPPING CLASS GROUPS

Vladimir Vershinin
Departement des Sciences Mathematiques, Universite Montpellier 2,
Place Eugene Bataillon, 34095 Montpellier cedex 5, France
Sobolev Institute of Mathematics,
Novosibirsk, 630090, Russia

vershini@math.univ-montp2.fr, versh@math.nsc.ru

Abstract

Pure braid groups and pure mapping class groups of a punctured sphere have many features in common. We consider the Lie algebra associated with the descending central series filtration of the pure braid group of a closed surface of arbitrary genus. R. Bezrukavnikov gave a presentation of this Lie algebra over the rational numbers. We show that his presentation remains true for this Lie algebra itself, i.e. over integers. We study also the graded Lie algebra of the descending central series of the pure mapping class group of a 2-sphere. A simple presentation of this Lie algebra is obtained.

TWO-GENERATED SUBGROUPS OF $\mathbf{PSL(2, C)}$ WHICH ARE EXTREMAL FOR JORGENSEN INEQUALITY AND ITS ANALOGUES

Andrey Vesnin
Sobolev Institute of Mathematics, Russia

vesnin@math.nsc.ru

Abstract

In the talk we will deal with discreteness conditions for two-generated subgroups of the isometry group of the hyperbolic 3-space. One of conditions is Jorgensen inequality in terms of traces of a generator and of a commutator of two generators. We will discuss groups which gives an equality in it. Some of them are related to hyperbolic knots and links.

FLAT ALMOST COMPLEX SURFACES IN THE NEARLY KÄHLER $S^3 \times S^3$

Luc Vrancken[1], B. Dioos, H. Li and H. Ma
[1]Université de Valenciennes, France
[1]luc.vrancken@univ-valenciennes.fr

Abstract

Nearly Kähler manifolds have been studies intensively in the 1970's by Gray. These nearly Kähler manifolds are almost Hermitian manifolds for which the tensor field ∇J is skew-symmetric. In particular, the almost complex structure is non-integrable if the manifold is non-Kähler. A well known example is the nearly Kähler 6-dimensional sphere, whose complex structure J can be defined in terms of the vector cross product on $\mathbb{R}^7$. Recently it has been shown by Butruille that the only homogeneous 6-dimensional nearly Kähler manifolds are the nearly Kähler 6-sphere, the nearly Kähler $S^3 \times S^3$, the projective space $\mathbb{C}P^3$ and the flag manifold $SU(3)/U(1) \times U(1)$. All these spaces are compact 3-symmetric spaces

There are two natural types of submanifolds of nearly Kähler (or more generally, almost Hermitian) manifolds, namely almost complex and totally real submanifolds. Almost complex submanifolds are submanifolds whose tangent spaces are invariant under J. Almost complex submanifolds in the nearly Kähler manifold S^6 have been studied by many authors. Also in the nearly Kähler $\mathbb{C}P^3$ some results have been obtained by Xu Feng.

In this talk we show how to study almost complex submanifolds of $S^3 \times S^3$. Compact 6-dimensional non-Kähler nearly Kähler manifolds do not admit 4-dimensional almost complex submanifolds, so the almost complex submanifolds are surfaces.

Previously, a classification of the totally geodesic almost complex surfaces has been ontained as well as the result that an almost complex immersion of a topological 2 sphere is necessary totally geodesic. In this lecture we will show how to obtain a classification, as well as explicit expressions of all almost complex flat surfaces.

CONNECTION BETWEEN SCREW SYSTEMS AND LINE CONGRUENCE

Şerife Nur Yalçın[1] **and Ali Çalışkan**[2]
[1,2]Science Faculty, Mathematic Department, Ege University, Turkey
[1]nurryalcin@gmail.com, [2]ali.caliskan@ege.edu.tr

Abstract

In this study, a third order screw system which compose of displacement of a line is considered by using Dimentberg's definition of pitch. After forming this system, the external bisectors of three different systems which are the elements of the screw systems are obtained. It is shown that these external bisectors are belong to the same recticongruence. Then the axis of this recticongruence is found. Finally, it is concluded that angles between the first, second, third screws and the axis of recticongruence are the same. In this way, it is shown that these screws are belong to the same line congruence.

AMS Subject Classification: 53A17, 53A25.

Key words: Line geometry, Screw system, Congruence.

HYPERBOLIC MONGE FORMS IN SENSE OF SLANT GEOMETRY

(joint work with Mikuri Asayama, Shyuichi Izumiya and Aiko Tamaoki)

Handan Yıldırım
Istanbul University, Turkey

handanyildirim@istanbul.edu.tr

Abstract

It is known that there are three kinds of pseudo-spheres in Lorentz-Minkowski space which are called Hyperbolic space, de Sitter space and lightcone. A basic duality theorem for four Legendrian dualities related with these pseudo-spheres was obtained in [3]. These Legendrian dualities were extended in [5] depending on a parameter $\phi \in [0, \pi/2]$. Moreover, as an application of these extended Legendrian dualities, one-parameter families of extrinsic differential geometries on hypersurfaces in Hyperbolic space were constructed in [2]. We call these geometries which include the results of [4] as a special case slant geometry of hypersurfaces in Hyperbolic space. In this talk, we first mention about the basic framework of slant geometry of hypersurfaces in Hyperbolic space. Then, we review the notion of Hyperbolic Monge form which was introduced in [4]. Finally, we give some examples of hypersurfaces in Hyperbolic space by means of Hyperbolic Monge forms in sense of slant geometry.

AMS Subject Classification: 53A35, 53B30, 57R45, 58C25, 58K99.

Key words: Legendrian dualities, Hyperbolic Monge forms, Hyperbolic space, Lorentz-Minkowski space.

References

[1] Arnol'd, V. I., Gusein-Zade, S. M. and Varchenko, A. N., *Singularities of Differentiable Maps,* Vol. I, Birkhäuser, 1985.

[2] Asayama, M., Izumiya, S., Tamaoki, A. and Yıldırım, H., *Slant geometry of spacelike hypersurfaces in Hyperbolic space and de Sitter space,* Revista Matemática Iberoamericana, **28(2)** (2012), 371-400.

[3] Izumiya, S., *Legendrian dualities and spacelike hypersurfaces in the lightcone,* Moscow Mathematical Journal, **9** (2009), 325-357.

[4] Izumiya, S., Pei, D-H. and Sano, T., *Singularities of hyperbolic Gauss maps,* Proceedings of the London Mathematical Society, **86** (2003), 485-512.

[5] Izumiya, S. and Yıldırım, H., *Extensions of the mandala of Legendrian dualities for pseudo-spheres in Lorentz-Minkowski space,* Topology and its Applications, **159** (2012), 509-518.

CARTAN'S CONNECTIONS IN A GENERALIZED FINSLER SPACE

Milan Lj. Zlatanović[1] and Milica D. Cvetković[2]
[1,2]University of Niš, Faculty of Science and Mathematics, 18000 Niš, Serbia
[2]College for Applied Technical Sciences, 18000 Niš, Serbia
[1]zlatmilan@pmf.ni.ac.rs, [2]milicacvetkovic@sbb.rs

Abstract

In this work we defined a generalized Finsler space ($\mathbb{GF_N}$) as $2N$-dimensional differentiable manifold with a non-symmetric basic tensor $g_{ij}(x,\dot{x})$, which applies that $g_{\underline{ij}\,\underset{\theta}{|}m}(x,\dot{x}) = 0$, $\theta = 1,2$. Based on non-symmetry of basic tensor, we obtained ten Ricci type identities, comparing to two kinds of covariant derivative of a tensor in Rund's sense. There appear two new curvature tensors and fifteen magnitudes, we called "curvature pseudotensors".

AUTHORS

Pablo Alegre,
Seville, Spain,
psalerue@upo.es

Borysenko Oleksandr Andriyovych,
Ukraine,
aborisenk@gmail.com

Bojana Anđelković,
Niš, Serbia,

Miroslava Antić,
Belgrade, Serbia,
mira@math.rs

Gülhan AYAR,
Yes, Turkey,
gulhanayar@gmail.com

M. Evren AYDIN,
Bucharest, Romania,

Murat Babaarslan,
Yozgat, Turkey,
murat.babaarslan@bozok.edu.tr

Vladimir Balan,
Bucharest, Romania,
vladimir.balan@upb.ro

Vitaly Balashchenko,
Belarus,
balashchenko@bsu.by

Yavuz Selim Balkan,
Düzce, Turkey,
y.selimbalkan@gmail.com

Gianluca Bande,
Cagliari, Italy,
gbande@unica.it

Cinthia Barrera,
Mexico,
aihtnic@matem.unam.mx

Ergin Bayram,
Samsun, Turkey,
erginbayram@yahoo.com

Cornelia-Livia Bejan,
Iasi, Romania,
bejanliv@yahoo.com

Burcu Bektaş,
Istanbul, Turkey,
bektasbu@itu.edu.tr

Momčilo Bjelica,
Zrenjanin, Serbia,
bjelica@tfzr.uns.ac.rs

D. E. Blair,
East Lansing, Michigan, USA,
blair@math.msu.edu

Beniamino Cappelletti-Montano,
Cagliari, Italy,
b.cappellettimontano@unica.it

Alfonso Carriazo,
Seville, Spain,
carriazo@us.es

Uday Chand De,
Calcutta, India,
uc_de@yahoo.com

Milica D. Cvetković,
Niš, Serbia,
milicacvetkovic@sbb.rs

Ali Çalişkan,
Turkey,
ali.caliskan@ege.edu.tr

Azime Çetinkaya,
Instambul, Turkey,
azzimece@hotmail.com

Simona Decu,
Bucharest, Romania,
simona.decu@gmail.com

Sezgin Altay Demirbağ,
Istanbul, Turkey,
saltay@itu.edu.tr

Ryszard Deszcz,
Wrocław, Poland,
Ryszard.Deszcz@up.wroc.pl

Antonio Di Scala,
Torino, Italy,
antonio.discala@polito.it

Ivko Dimitrić,
Fayette, Pennsylvania, USA,
ivko@psu.edu

Ivan Dimitrijevic,
Belgrade, Serbia,
ivand@matf.bg.ac.rs

B. Dioos,

Branko Dragovich,
Belgrade, Serbia,
dragovich@ipb.ac.rs

Irem KUPELİ ERKEN,
Bursa, Turkey,
iremkupeli@uludag.edu.tr

Nikolay Yurievich Erokhovets,
Moscow, Russian Federation,
erochovetsn@hotmail.com

Luis M. Fernández,
Seville, Spain,
lmfer@us.es

Anatoly Fomenko,
Moscow, Russia,
atfomenko@mail.ru

Tatiana Fomenko,
Moscow, Russia,
tn-fomenko@yandex.ru

Fabio Gavarini,
Roma, Italy,
gavarini@axp.mat.uniroma2.it

Małgorzata Głogowska,
Wrocław, Poland,
Magorzata.Glogowska@up.wroc.pl

Dara Gold,
Boston, USA,
daracaseygold@gmail.com

Milica Grbović,
Kragujevac, Serbia,
milica_grbovic@yahoo.com

Jelena Grujic,
Belgrade, Serbia,
jelenagg@gmail.com

Dmitry V. Gugnin,
Moscow, Russia,
dmitry-gugnin@yandex.ru

Sinem Güler,
Istanbul, Turkey,
singuler@itu.edu.tr

Graham Hall,
Aberdeen, United Kingdom,
g.hall@abdn.ac.uk

Irena Hinterleitner,
Brno, Czech Republic,
hinterleitner.irena@seznam.cz

Stefan Ivanov,
Sofia, Bulgaria,
ivanovsp@fmi.uni-sofia.bg

Slavik Jablan,
Belgrade, Serbia,
slavik.jablan@ict.edu.rs

Włodzimierz Jelonek,
Cracow, Poland,
wjelon@pk.edu.pl

Nenad Jovanović,
Niš, Serbia,
nelejovanovic@yahoo.com

Georgios Kaimakamis,
Attiki, Greece,
gmiamis@gmail.com

E.Kantonistova,
Moscow, Russia,
kysin@rambler.ru

Emin Kasap,
Samsun, Turkey,
kasape@omu.edu.tr

Kamran Khan,
Aligarh, India,
kamran35d@gmail.com

Bahar Kirik,
Istanbul, Turkey,
bkirik@itu.edu.tr

Miljan Knežević,
Belgrade, Serbia,
kmiljan@matf.bg.ac.rs

Oldrich Kowalski,
Praha, Czech Republic,
kowalski@karlin.mff.cuni.cz

Sonja Krasić,
Niš, Serbia,
sonja.krasic@gaf.ni.ac.rs

Hristina Krstić,
Niš, Serbia,
hristinaa@hotmail.com

Jan Kurek,
Lublin, Poland,
kurek@hektor.umcs.lublin.pl

Miroslav Kures,
Brno, Czech Republic
kures@fme.vutbr.cz

Irina Kuzmina,
Kazan, Russia,
iranina@mail.ru

H. Li,

Tee How Loo,
Kuala Lumpur, Malaysia,
looth@um.edu.my

H. Ma,

Pradip Majhi,
Darjeeling, West Bengal, India,
mpradipmajhi@gmail.com

Hristo Manev,
Plovdiv, Bulgaria,
ico_manev@yahoo.com

Mancho Manev,
Plovdiv, Bulgaria,
mmanev@uni-plovdiv.bg

Veronica Martin-Molina,
Zaragoza, Spain,
vmartin@unizar.es

Nikolay Nikolaevich Martynchuk,
Russian Federation,
mnick45@bk.ru

Koji Matsumoto,
Yonezawa, Yamagata, Japan,
tokiko_matsumoto@yahoo.com

Monica Merkle,
Rio de Janeiro, Brasil,
monica@im.ufrj.br

Adela MIHAI,
Bucharest, Romania,
adela_mihai@fmi.unibuc.ro

Ion Mihai,
Bucharest, Romania,
imihai@fmi.unibuc.ro

Josef Mikeš,
Olomouc, Czech Republic,
josef.mikes@upol.cz

Włodzimierz Mikulski,
Cracow, Poland,
wjelon@pk.edu.pl

Dmitry Millionshchikov
Moscow, Russia
mitia_m@hotmail.com

Vuk Milošević,
Niš, Serbia,
vukamer@yahoo.com

Ivan Minchev,
Sofia, Bulgaria,
minchevim@yahoo.com

Svetislav Minčić,
Niš, Serbia,
svetislavmincic@yahoo.com

Andrey Mironov,
Moscow, Russia,
mironov@math.nsc.ru

Nataša Ž. Mišić,

Emil Molnár,
Budapest, Hungary,
emolnar@math.bme.hu

M.N. Mukherjee,
Kolkata, West Bengal, India,
mukherjeemn@yahoo.co.in

Marian Ioan Munteanu,
Iasi, Romania,
marian_ioan_munteanu@yahoo.com

Pegah MUTLU,
Istanbul, Turkey,
sariaslani@itu.edu.tr

Stanislav Nikolaenko,
Moscow, Russia,
nikostas@mail.ru

Vladan Nikolić,
Niš, Serbia,
vladan_nikolic@yahoo.com

Ana Irina Nistor,
Iasi, Romania,
ana.irina.nistor@gmail.com

Zbigniew Olszak,
Wroclaw, Poland,
zbigniew.olszak@pwr.wroc.pl

Füsun ÖZEN ZENGİN,
Istanbul, Turkey,
fozen@itu.edu.tr

Oscar Palmas,
Mexico,
oscar.palmas@ciencias.unam.mx

Konstantina Panagiotidou,
Thessaloniki, Greece,
kapanagi@gen.auth.gr

Anica Pantić,
Kragujevac, Serbia,
anica.pantic@kg.ac.rs

Petar Pejić,
Niš, Serbia,
petar.pejic@gaf.ni.ac.rs

Alexander Petkov,
Sofia, Bulgaria,
a_petkov_fmi@abv.bg

Zoran Petrović,
Belgrade, Serbia,
zoranp@matf.bg.ac.rs

Alicia Prieto-Martin,
Seville, Spain,
aliciaprieto@us.es

Mileva Prvanović,
Belgrade, Serbia,

Ljiljana Radović,
Niš, Serbia,
ljradovic@gmail.com

Zoran Rakić,
Belgrade, Serbia,
zrakic@matf.bg.ac.rs

Gabriel Ruiz,
Mexico,
gruiz@matem.unam.mx

Zerrin ŞENTÜRK,
Instambul, Turkey,
senturk@itu.edu.tr

Mohammad Shuaib,
Aligarh, India,
Shuaibyousuf6@gmail.com

Mića S. Stanković,
Niš, Serbia,
stmica@ptt.rs

Sergey Stepanov,
Moscow, Russian Federation,
s.e.stepanov@mail.ru

Elena Stepanova,
Olomouc, Czech Republic,
stephelena@list.ru

Jelena Stojanov,
Novi Sad, Serbia,
stojanov.jelena@gmail.com

Milica Stojanović,
Belgrade, Serbia,
milicas@fon.rs

Marko Stošić,
Belgrade, Serbia,
mstosic@isr.ist.utl.pt

Jenő Szirmai,
Budapest, Hungary,
szirmai@math.bme.hu

Tijana Šukilović,
Belgrade, Serbia,
tijana@matf.bg.ac.rs

Kostadin Trenčevski,
Skopje, Macedonia,
kostatre@pmf.ukim.mk

Irina Tsyganok,
Moscow, Russian Federation,
i.i.tsyganok@mail.ru

Alexey Avgustinovich Tuzhilin,
Moscow, Russian Federation,
tuz@mech.math.msu.su

Vladimir Vershinin,
Montpellier, France,
Novosibirsk, Russia,
vershini@math.univ-montp2.fr
versh@math.nsc.ru

Nenad O. Vesić,
Niš, Serbia,
vesic.specijalac@gmail.com

Andrey Vesnin,
Russia,
vesnin@math.nsc.ru

Luc Vrancken,
Valenciennes, France,
luc.vrancken@univ-valenciennes.fr

Şerife Nur Yalçin,
Turkey,
nurryalcin@gmail.com

Yusuf Yayli,
Ankara, Turkey,
yayli@science.ankara.edu.tr

Handan Yıldırım
Istanbul, Turkey,
handanyildirim@istanbul.edu.tr

Milan Lj. Zlatanović,
Niš, Serbia,
zlatmilan@pmf.ni.ac.rs